Visualization and Quantity Estimation of Mismatch Gamuts of Metamers viewed under different Illuminants

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ABSTRACT
This paper deals with the effect of illuminant metamerism. A method is proposed which characterizes the mismatch gamut of all reflective metamers for a set of illuminants which are viewed under a different illuminant. The mismatch gamut is described by a Metamer Boundary Descriptor (MBD) matrix in the CIE L*a*b* color space which is calculated in the CIE XYZ color space using linear programming . The MBD matrix can be used to visualize the mismatch gamut and to calculate it's volume in the CIE L*a*b* color space. A closed formula to determine the volume using the MBD matrix will be presented. Results of simulation experiments for various illuminants and reflection spectra will be given.

1. INTRODUCTION

Reflective metamers are pairs of color stimuli with equal tristimulus values for a set of illuminating illuminants but with different reflection spectra. For an illuminant which is not in the set of the mentioned illuminants the reflection spectra result in different tristimulus values. To determine the theoretical magnitude of color mismatches a Monte-Carlo method [1] or linear programming [2] have been used.

In this paper we propose a method to characterize the mismatch gamut boundaries in the CIE L*a*b* color space by calculating a Metamer Boundary Descriptor (MBD) matrix. This MBD matrix approximately describes the mismatch gamut by storing its boundary points in every entry. To calculate the entries of the MBD we use a priori knowledge about the physics of natural reflectance spectra and also a linear programming technique [3].

Using the MBD the volume of the mismatch gamut can be easily calculated in the nearly perceptual uniform CIEL*a*b* color space.

2. METHOD

In the following text we describe the problem in a discrete formulation by sampling all spectra in N equal wavelength intervals in the range of [400 nm, 700 nm]. The tristimulus values of a reflection spectrum r for a illuminant l are given by

$(X,Y,Z)^T = \Omega_l r$ where $\Omega_l = \begin{pmatrix} l_1 \cdot \bar{x}_1 & \cdots & l_N \cdot \bar{x}_N \\ l_1 \cdot \bar{y}_1 & \cdots & l_N \cdot \bar{y}_N \\ l_1 \cdot \bar{z}_1 & \cdots & l_N \cdot \bar{z}_N \end{pmatrix}$ and the color matching functions $\bar{x}, \bar{y}, \bar{z}$.

Two reflection spectra r_1, r_2 $(r_1 \neq r_2)$ are called metamer for a set of illuminants $l_1, \ldots, l_M$ if they apply

$$\Omega_{l_i} r_1 = \Omega_{l_i} r_2 = (X_i, Y_i, Z_i)^T,\ i = 1, \ldots, M$$

For a different illuminant L they might not match, i.e.

$$\Omega_L r_1 \neq \Omega_L r_2$$

To determine all tristimulus values which occur under the illuminant L for metamers relating to the set of illuminants $l_1, \ldots, l_M$ we calculate the boundary of the mismatch gamut using a Metamer Boundary Descriptor (MBD).

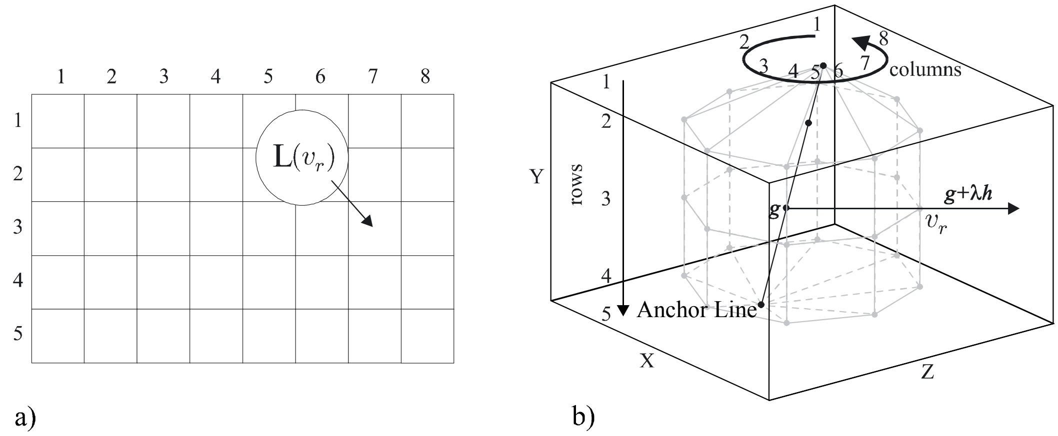

Figure 1: a) The MBD matrix with $n = 5$ and $m = 8$ **b)**. The corresponding metamer subspace M_{XYZ} in CIE XYZ color space. Each entry of the MBD matrix is sampled in the CIE XYZ color space using a linear programming technique. The sampled point v_r is transformed in the CIE L*a*b* color space and stored in the MBD matrix.

In the following we denote a mismatch gamut in the CIE XYZ color space by M_{XYZ} and the corresponding gamut in CIE L*a*b* color space by M_{Lab} (we omit the symbols of the participate illuminants for clarity). The transformation from CIE XYZ to CIE L*a*b* is denoted by L, so $\mathrm{L}(M_{XYZ}) = M_{Lab}$ applies.

The MBD is a $n \times m$ matrix which stores a boundary point of the mismatch gamut M_{Lab} in every entry (see Figure 1a). Each row contains m contour points of the set for a fixed L* value. The boundary points will be calculated in the CIEXYZ space using the following linear programming (LP) problem, which samples the metamer subspace along a straight line $g + \lambda v$:

Solve: $-\lambda = \textbf{min}$

using the linear constraints: $r \geq 0$, $r \leq 1$, $Hr \leq p$, $-Hr \leq p$, $\Omega_{l_i} r = (X_i, Y_i, Z_i)^T, i = 1,\ldots,M$,

$$\Omega_L r = v_r, g + \lambda v = v_r, \lambda \geq 0$$

Constraint $r \geq 0$ ensures the positivity of reflectance spectra and constraint $r \leq 1$ guarantees the boundness for non-fluorescent surfaces. In addition, we use the smoothness constraint $Hr \leq p$ and $-Hr \leq p$ with a smoothing parameter $p > 0$ and a convolution matrix H to apply the Laplace operator to r. Constraint $\Omega_{l_i} r = (X_i, Y_i, Z_i)^T, i = 1,\ldots,M$ ensures that we only take into account reflectance spectra *r which* are metamers relating to the set of illuminants. Constraint $\Omega_L r = v_r$ introduces the considered illuminant *L* with the auxiliary variable v_r which is necessary for the sampling constraint $g + \lambda v = v_r$. This constraint together with the objective function ($-\lambda$ = min) allows us to sample the boundary of the mismatch gamut in the CIE XYZ space along the mentioned line (see Figure 1b). After solving this LP we get a tristimulus value v_r, which is the intersection of the line and the boundary of the mismatch gamut and the appropriate reflection spectrum *r* as results.
For each boundary point the parameters *g* (anchor point) and *v* (direction vector) defining the sampling line have to be chosen in a way that the MBD entries are uniformly distributed on the boundary of the mismatch gamut in the CIE L*a*b* color space. A special technique for choosing these parameters is explained in [3].
The MBD matrix represents a wired grid model of the corresponding mismatch gamut, so it is suitable for visualization, e.g. using Matlab [4] (see Figure 2).

3. VOLUME OF THE MISMATCH GAMUT

The volume of a mismatch gamut M_{Lab} in the CIE L*a*b* color space is given by

$$Vol(M_{Lab}) = \int_{M_{Lab}} 1 \; dL^* da^* db^*$$

Using different substitutions the integral above can be transformed in

$$Vol(M_{Lab}) = \int_{Y_{\min}}^{Y_{\max}} \int_0^{2\pi} \int_0^{\mathrm{R}(Y,\phi)} R \left| \det[\mathrm{DL}(X(R\cos(\phi),Y),Y,Z(Y,R\sin(\phi)))] \right| dR\, d\phi\, dY$$

where $\mathrm{L}(X,Y,Z)$ is the transformation from CIE XYZ to CIE L*a*b*, DL is the corresponding Jacobian matrix, $X(X_0,Y)$, $Z(Z_0,Y)$ translate the origin to a point on the anchor line (see Figure 1b), $Y_{\min}$, $Y_{\max}$ are the minimal and the maximal Y values of M_{XYZ} and $\mathrm{R}(Y,\phi)$ is a function which determines the distance of the boundary point of M_{XYZ} from the anchor point. $\mathrm{R}(Y,\phi)$ is implicitly given for the entries of the MBD matrix end can be determined in the following manner: Every MBD matrix entry can be written as follows in the CIE XYZ color space

$$a_{i,j} = \begin{bmatrix} X_{i,j} \\ Y_{i,j} \\ Z_{i,j} \end{bmatrix} = \begin{bmatrix} R_{i,j}\cos(\phi_{i,j}) \\ 0 \\ R_{i,j}\sin(\phi_{i,j}) \end{bmatrix} + g_i$$

where $g_i = (g_{i,X}, g_{i,Y}, g_{i,Z})^T$ is the anchor point of all sampling lines related to the *i*th row.

The $Y_i := g_{i,Y}$, $\phi_{i,j}$ and $R_{i,j}$ describe sheared cylindrical coordinates and can be stored in a matrix

$$\begin{bmatrix} (Y_1,\phi_{1,1},R_{1,1})^T & \cdots & (Y_1,\phi_{1,m},R_{1,m})^T \\ \vdots & & \vdots \\ (Y_n,\phi_{n,1},R_{n,1})^T & \cdots & (Y_n,\phi_{n,m},R_{n,m})^T \end{bmatrix}$$

Using a decomposition $0 = R_{i,j,1} < \ldots < R_{i,j,K} = R_{i,j} = \mathrm{R}(Y_i,\phi_{i,j})$ we can approximate the integral by a sum

$$Vol(M_{Lab}) \approx \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{K} R_{i,j,k} \left| \det[\mathrm{DL}(X(R_{i,j,k}\cos(\phi_{i,j}),Y_i),Y_i,Z(Y_i,R_{i,j,k}\sin(\phi_{i,j})))] \right| \Delta R_{i,j,k} \Delta\phi_{i,j} \Delta Y_i$$

If the MBD matrix has been calculated initially the above sum describes a closed formula to calculate the volume of the mismatch gamut.

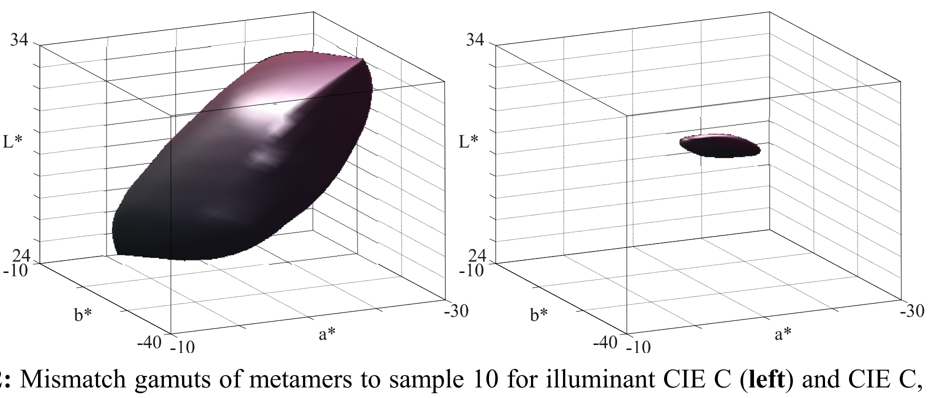

Figure 2: Mismatch gamuts of metamers to sample 10 for illuminant CIE C (**left**) and CIE C, CIE A (**right**) viewed under illuminant CIE F2.

4. RESULTS

We have calculated the volumes of various mismatch gamuts of metamers which match to the 24 reflection spectra of a MacBeth Color Checker under one or two standard CIE illuminants and are viewed under a different CIE illuminant. The involved CIE illuminants are CIE A (incandescent light), CIE C (average daylight,) and CIE F2 (cool white fluorescent). The MBD matrix dimension has been set to 8×8. All spectra have been sampled in 5 nm wavelength intervals. The smoothness parameter p has been set to 0.0035. The results are presented in Table 1.

Table 1: Volumes of metamer mismatch gamuts in the CIE L*a*b* color space for metamers which match the reflection spectra of the MacBeth Color Checker for one or two illuminants and are viewed under an different illuminant.

Viewing illum.	**A**	**A**	**C**	**C**	**F2**	**F2**
Metamer match illum.	**C**	**C, F2**	**A**	**A, F2**	**A**	**A, C**
Sample 1	22.5140	0.1527	27.8000	0.1265	1012.0000	5.4548
Sample 2	1.9837	0.0000	2.9828	0.0000	105.9200	0.0456
Sample 3	6.2987	0.0427	5.9226	0.0242	279.9000	1.1886
Sample 4	15.8150	0.1342	18.2280	0.0859	610.8300	3.5794
Sample 5	3.3351	0.0140	4.0462	0.0077	155.7000	0.3284
Sample 6	0.6674	0.0043	0.8821	0.0028	35.2140	0.0948
Sample 7	2.3543	0.0000	3.4754	0.0000	127.3200	0.1778
Sample 8	10.3250	0.0456	7.7043	0.0181	425.5300	0.6763
Sample 9	3.0434	0.0000	3.4987	0.0000	119.1400	0.0000
Sample 10	30.3760	0.2204	31.9710	0.1538	1534.2000	7.4879
Sample 11	0.5679	0.0000	0.6110	0.0000	13.2230	0.0000
Sample 12	1.5618	0.0087	2.1819	0.0078	85.5670	0.1342
Sample 13	17.8910	0.0122	11.4620	0.0150	649.5600	0.5963
Sample 14	2.2879	0.0000	3.5323	0.0000	115.7700	0.0433
Sample 15	5.6815	0.0000	5.7598	0.0000	160.6700	0.0000
Sample 16	0.4156	0.0000	0.6436	0.0000	15.9410	0.0000
Sample 17	1.5681	0.0000	1.6485	0.0000	52.7540	0.0000
Sample 18	3.3320	0.0118	3.2222	0.0037	164.0000	0.2249
Sample 19	0.1933	0.0014	19.5640	1.2764	8.9936	0.0459
Sample 20	0.7579	0.0050	1.1395	0.0036	40.6350	0.1476
Sample 21	2.0089	0.0135	2.9332	0.0098	108.3600	0.4008
Sample 22	6.6028	0.0447	8.2961	0.0305	339.1000	1.3695
Sample 23	27.0170	0.1932	27.7110	0.1248	1217.6000	6.1978
Sample 24	64.9590	1.3170	61.5000	0.8674	2385.9000	41.5850

5. CONCLUSIONS

We presented a method which characterises the mismatch gamuts in the CIE L*a*b* color space of metamers for a set of illuminants which are viewed under a different illuminant. The method uses a linear programming technique and a priori knowledge about natural reflection spectra to sample the boundary of the mismatch gamut and to store the sampling colors which are transformed in the CIE L*a*b* color space in a metamer boundary descriptor matrix. This matrix can be used to visualise the mismatch gamut and to approximate its volume.

References

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