Fast Low-Level Filter Systems for Multispectral Color Images

R. Lenz[1], T. H. Bui[1], K. Takase[2]
[1]Department of Science and Technology, Linköping University,SE-60174 Norrköping (Sweden)
[2]Department of Information and Image Sciences, Chiba University, Chiba 263-8522 (JAPAN)
Corresponding author: R. Lenz (reile@itn.liu.se)

ABSTRACT

The investigation of low-level filter systems for color image processing is of interest in many different areas of color signal processing, computational color vision and the understanding of biological color vision systems. In color signal processing they are useful for image compression, they are basic building blocks for computational color vision systems and the investigation of their general properties may be useful in understanding basic principles behind evolutionary systems. Here we describe the following aspects: (1) the connection to the theory of group representations, (2) the relation between the symmetry properties of a stochastic process and its principal component analysis, (3) their algorithmic derivation with the help of software packages like Maple, Mathematica and Matlab, (4) their relation to the investigation of natural scene statistics, (5) their fast FFT-like implementations (also on special hardware), (6) some applications where fast executions are necessary. We illustrate their properties with some application examples from the image database retrieval.

1. INTRODUCTION AND MOTIVATION

The design of low-level filter systems (especially edge and line filters, corner detectors etc.) has a long tradition in image processing. The majority of investigations deal with grey-value images and many are based on heuristic design principles. Color filters on the other hand are often simple generalizations of the corresponding grey-value systems and are often limited to the special case of RGB images with their three-dimensional geometry. Currently it is therefore fair to say that the design of low-level color filters is still an open research problem. At the same time one can also see an increasing need for efficient color filter systems. Some examples are the following: (1) Digital cameras replace analogue cameras and the digital image processing capabilities of the chips in these cameras is getting more powerful with every new generation. (2) The number of digital images is growing at a very fast rate; both in private collections, image databases and on websites. Efficient multimedia processing of large amounts of image data requires very fast processing methods. (3) The traditional three-dimensional description of color information in the form of RGB images is too restrictive for many high-end applications. A current trend is therefore to enhance the color imaging capabilities of cameras and scanners by adding more color channels. A general approach to low-level filter design should therefore not be restricted to three-dimensional color representations but should generalize naturally to the processing of multi-channel images.

2. SYMMETRY-BASED FILTER DESIGN PRINCIPLES

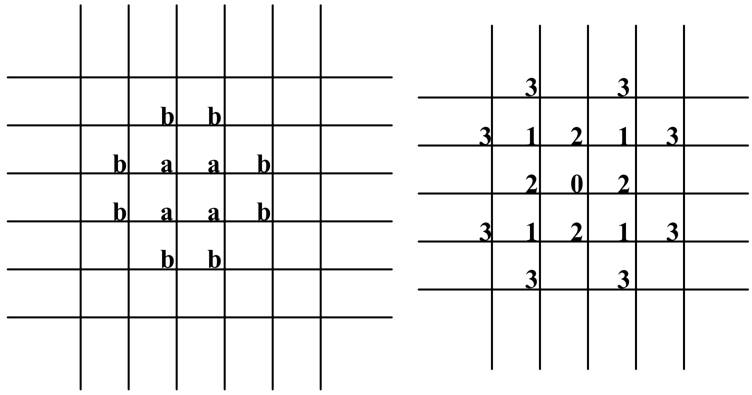
Consider an array of pixels and assume that at each position we have the same number of color channels (one for black-white cameras, three for RGB sensors and N for a general multi-channel detector). We also assume that the pixels are spatially organized on a regular geometric pattern (typically on a square grid but hexagonal sampling schemes are also common). Given this basic detector arrangement we have to decide how to combine and process output signals from neighbouring detectors (see Fig. 1 for the square grid case).

In the following we will investigate in the framework of stochastic processes. We model the values at M pixel positions with N channels each by a vector s of size MxN.

We also make the following assumptions about the stochastic properties of these vectors:

We assume that all color channels are interchangeable. By this we mean that for any permutation π of the color channels we find that the original vector s and its channel-permuted version have the same probability. We say that the group **S(N)** of all permutations of N elements is a symmetry group of the stochastic process.

We also assume that the M points are located on a grid and that this grid is closed under a group of geometrical transformations. In the case of a square grid we assume that these transformations are the rotations of 0, 90, 180 and 270 degrees and the reflections on the diagonals. These transformations form the dihedral group **D(4)**. Also here we require that for all the symmetry transformations σ in **D(4)** the vector s and its rotated and reflected versions have the same probability. The dihedral group is a symmetry group of the stochastic process. Combining both assumptions the stochastic process is symmetric with respect to the permutation group **S(N)** and the dihedral group **D(4)**, The case of stochastic processes with dihedral symmetries was investigated earlier[1] and the interested reader can find a description of the basic tools from the theory of group representations and the main results related to the geometrical properties of the grid there.



Even-sized neighborhoods *Odd-sized neighborhoods*

Figure 1: Neighborhoods on square grid

For a stochastic process described by vectors s we denote the correlation matrix by $\boldsymbol{C}$. It is easy to see that all the combinations (π,σ) of channel permutations π and geometrical operations σ can be described by matrix operations $T(\pi,\sigma)$ on the vectors s. The assumption that all these operations have the same probability implies that the correlation matrix $\boldsymbol{C}$ satisfies the equations

$$C = T(\pi,\sigma)^{-1} C T(\pi,\sigma) = T(\pi,\sigma)^{T} C T(\pi,\sigma) \tag{1}$$

The matrices $T(\pi,\sigma)$ form a group and the investigation of matrices $\boldsymbol{C}$ that satisfy the conditions in the Eq. (1) is one of the topics in the theory of group representations. A detailed description of these results can be found in the literature[1,2,3] and here we will only summarize some of the results:

1. Geometrically every neighbourhood can be split into a collection of points, so called orbits, where each orbit consists of one, four or eight points (corresponding to collections of points like those marked 0, 1, 3 in Fig. 1). In a first step it is only necessary to define filter systems defined on those orbits.
2. For all correlation matrices of size $(MxN)^2$ that satisfy Eq. (1) it is possible to find a basis in the (MxN)-dimensional space such that the correlation matrix is block-diagonal in the new coordinate system.
3. The basis depends only on the symmetry group and not on the particular matrix $\boldsymbol{C}$. The block structure is thus defined by the symmetry group only. Algorithms from representation theory can be used to construct the basis and those basis vectors define the low-level filter system.
4. In the context of low-level color and multispectral image processing the previous result shows that given the sampling scheme of the sensor (image) and the number of spectral channels it is possible to perform a partial Principal Component Analysis (PCA) of all stochastic processes that share the same symmetry group.
5. The bases used in PCA are usually orthonormal. If we relax the requirement of orthonormality then it can be shown that it is possible to select equivalent bases (filter systems) consisting of coefficients 1, -1 and 0 only. These filter systems can therefore be implemented using additions and subtractions exclusively. Normalization can be performed after the basic filtering in the cases where this is necessary.

6. The goal of the filter design strategy used here is NOT to construct single filter functions but to construct complete filter systems. By this we mean a lossless filter system that recodes the original (MxN)-dimensional multispectral data distribution in a new coordinate system. The tools from group representation theory construct such filter systems and it can be shown that the filter systems obtained by these methods have an inherent structure that allows fast implementations. This is achieved by re-using partial results in the computation of several filter results. The relation between the straightforward implementation and their fast versions is similar to the relation between the Discrete Fourier Transform (DFT) and the Fast Fourier Transform (FFT).

Up to now we motivated the filter design by their PCA-related approximation properties. There is however another symmetry related approach. The basic idea behind this construction is the observation that if two color distributions on the sensor are related by a symmetry operation (like a rotation or a channel permutation) then the filter output for these two distributions should also be related. A typical motivation is the argument that we would like to recognize patterns independent of their orientation or permutations of the color channels. Such invariance principles have been popular in pattern recognition, computer vision and investigations of biological vision systems and it can be shown that this approach leads to exactly the same filter systems.

3. SOME EXAMPLES

We will now illustrate some of the general results with some examples obtained in the design of filter systems for square-grid three-channel images (such as the standard RGB images). We saw above that the basic building blocks in this case consist of 1, 4 or 8 points with 3 channels per point. The corresponding vectors have thus length 3, 12 and 24. The general theory for 3-channel images on square grids gives the following results:

1. There exist 15 different types of filter systems
2. These basic filter systems consist of 1, 2 or 4 filter functions

Applied to a 12-dimensional vector belonging to a corner orbit consisting of four points with 3 channels each (such as those marked with the symbols **a** or **1** in Fig. 1) the filter system splits the 12-dimensional space into components of size 1,2,1,2,2,4. The structure of this 12-dimensional space in terms of the 15 basic filter types is shown in Table 1:

Table 1: Decomposition of the 12-dimensional signal space (4-point corner orbit with 3 color channels)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	2	0	0	0	1	0	2	0	0	0	2	0	4

An overview over the results of applying all 12 filter functions for this orbit type to the Pepper image is given in Fig. 2. The first function is an averaging filter (all-one coefficients). The second and third filters are R-B and G-B filters. In this simple case the filters are combinations of the spectral combinations R+G+B, R-B, G-B combined with four spatial filters. If we denote the four points in the corner orbit by p_1, p_2, p_3, p_4 then the automatic derivation of these four filter functions resulted in the following four filter functions:

$$f_1 = p_1 + p_2 + p_3 + p_4; \quad f_2 = p_1 - p_2 + p_3 - p_4; \quad f_3 = p_1 - p_3; \quad f_4 = p_2 - p_4; \qquad (2)$$

Recombining them we see that this is equivalent to an averaging filter, an x- and a y-gradient filter plus a diagonal filter. We see immediately that the partial sums p_1+p_3 and p_2+p_4 can be reused both in the computation of f_1 and of f_2. Analysing the structure of this filter system shows that all 12 filter shows that all 12 filter results can be computed by 34 additions/subtractions, no multiplications are necessary. This highly structured and very efficient structure of the filter systems makes them especially suitable for implementation on fast, special image processing hardware (GPU's) where the filter results can be computed at the rates that makes them attractive for video-processing and processing of very large number of images for example in image databases. The computation time when using a Geforce6600 GT GPU was less than 0.0056 second (180FPS) for computing all twelve filter results for an image of size 512^2.

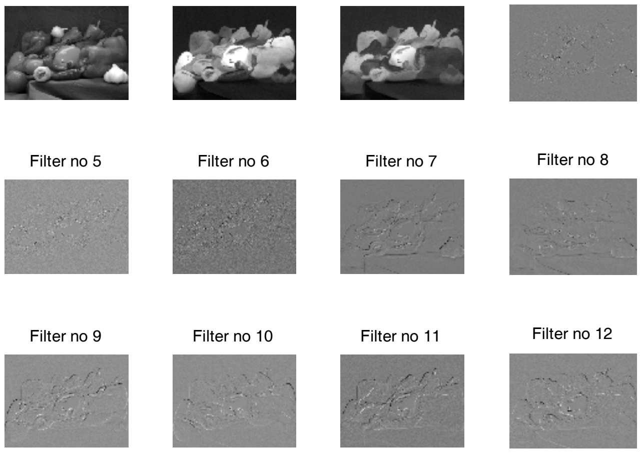


Figure 2: Corner orbit filter images of Peppers

In the beginning we motivated the interest in these filter systems by arguing that they are especially suited for stochastic processes with symmetry groups. It is therefore of interest to test how far the required symmetry conditions fulfilled in realistic applications. In Fig. 4 we show the result of one such test where we randomly selected 10.000 images from a large database with more than 750.000 images. From each image we selected 2 random patches of size 16x16 pixels. Since every pixel consists of the usual RGB channels we get signal vectors of dimension 16x16x3=768. Fig. 3 shows both, the original correlation matrix and the transformed block-diagonal correlation matrix of the same dataset but now computed in the new group-theoretically constructed basis. In the transformed correlation matrix we scaled down the dominating part in the upper left corner (related to the averaging filters) to visualize the structure of the rest of the matrix.

4. CONCLUSIONS

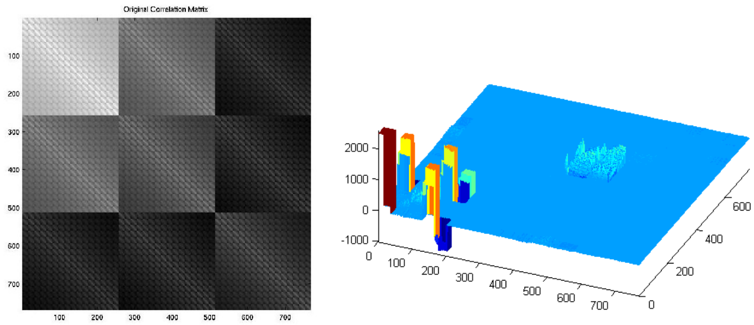


Figure 3: Original Correlation matrix (left) and transformed correlation matrix (right)

We generalized the symmetry related filter design strategy from purely geometrically defined spatial symmetries to the case of spatio-spectral images. We investigated the properties of low-level image processing filter systems that are based on the symmetries of the sampling grid and the permutation invariance of the color channels. We showed that the resulting filter systems have interesting invariance properties, compute a partial PCA and allow very fast implementations that make them interesting candidates for hardware implemented filter systems and for applications that require the fast filtering of very large amount of images.

Acknowledgements: The financial support from Vinnova (the Swedish Agency for Innovation Systems), the Swedish Research Council and STINT (The Swedish Council for International Cooperation in Research and Higher Education) is gratefully acknowledged. The image database was provided by Matton AB in Stockholm.

References

1 R. Lenz "Investigation of receptive fields using representations of dihedral groups," *J. Vis. Comm. Im. Rep.*,vol. 6, pp. 209–227, 1995.

2. J. P. Serre, *Linear Representations of Finite Groups*. Springer, 1977.

3. E. Stiefel and A. Fässler, *Gruppentheoretische* Methoden und ihre Anwendungen, Teubner, 1979