# Practical Considerations for Calculating CIE Tristimulus Values

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## ABSTRACT

Weighting tables are widely used for calculating tristimulus values in industrial applications. The CIE (International Commission on Illumination) has never provided guidelines to calculate weighting tables, and various discrepant methods have been used. Hence it is possible to obtain significantly different tristimulus values from the same set of spectral data. Two main practical issues are considered here. The first one is which kind of weighting tables should be used? The second one is how the weighting tables should be prepared for a shorter wavelength range than the one recommended by CIE?

## 1. INTRODUCTION

CIE defined the tristimulus values in terms of integrations. Since the functions involved have no analytical expressions and only some discrete function values are available through measurements, the numerical integration thus must be used. Although CIE recommended the numerical integration is the simple summation of the products of the function values sampled at 1 nm interval, industrial applications prefer using the weighting tables at a larger, 10 or 20 nm for example, interval prepared in advance and the summations are carried out by summing all the products of the weights and the measured reflectance values $R_i^{'}$ at the corresponding wavelength interval. Furthermore, the CIE recommended the visible range is from 380 nm to 780 nm for the non-fluorescent applications, the industry applications prefer the range of 400 nm to 700 nm since most of the instruments provide data within this range.

CIE has never provided guidelines to calculate the weighting tables, and various discrepant methods have been used. Hence it is possible to obtain significantly different tristimulus values from the same set of spectral data. For overcoming this problem, the ASTM Intl. [1] has published two sets of weighting tables known as Table 5 and Table 6 respectively. The weighting tables of Table 5 must be used with the reflectance corrected using the Stearns and Stearns [2] (SS) method, and weighting tables of Table 6 must be used with the reflectance without the SS correction. However in practice, the illuminant required may be different from the standard and users have to prepare their own weighting tables corresponding to the illuminant actually used. ASTM Intl. E2022-99 [3] provided a standard method to calculate weighting tables of Table 5 for a given non-standard illuminant. Recently, direct methods [4,5] were developed for generating the optimum and least squares weighting tables. Numerical comparisons have shown that the optimum and least square weighting tables are more accurate than the ASTM Intl. weighting tables of Tables 5 and 6.

Another problem associated with practical applications is that reflectance values $R_i^{'}$ are measured at 10 or 20 nm interval between 400 and 700 nm. Thus, the visible range (400 nm,700 mm) is shorter than the range (380 nm, 780 mm) recommended by CIE [6] and the range (360 mm,780 mm) provided by ASTM Intl. [1]. CIE [7] and ASTM Intl.[1] recommended to "set the missing values to the nearest measured value", which is called the **E1** method in this study. Now CIE [6] has recommended that the missing values can be linearly extrapolated from the nearest two data points, which is called the **E2** method here. However, there is no evidence reported to show the linear (**E2**) extrapolation is better. Besides, in many occasions the 10-nm or 20-nm weighting tables were built by selecting directly from 1-nm spectral power distribution and colour matching functions every 10-nm or 20-nm. Certainly this is not a proper way to do it. However, there are no quantitative comparisons between ASTM Intl. standard tables and direct selection weighting tables. The aims of this study are twofold.

Firstly we want to know how large colorimetric errors may be introduced when each of the two extrapolation methods (**E1** and **E2**) is used under a particular weighting table. Secondly, we want to know how large errors may be introduced when different weighting tables are used.

## 2. EXPERIMENTAL DESIGN

The 1nm reflectance values [8] measured from 1269 matt Munsell color chips between 380 nm and 800 nm are used. The missing values between 360 nm and 380 nm were obtained using the linear extrapolation and the values beyond 780 nm were simply omitted. Finally, the set of reflectance values at 1 nm interval between 360 nm and 780 nm was used as a standard 1nm reflectance set. Note that the reason to use (360 nm, 780 nm) rather than (380 nm, 780 nm) as the wavelength range is that we want to use the ASTM Intl. standard weighting tables of Tables 5 and 6.

The 10-nm and 20-nm reflectance functions of each samples between 400 nm and 700 nm were calculated from the standard 1 nm reflectance set by assuming an ideal instrument [4,5]. This is to simulate the practical measurements. Subsequently the "measured" reflectance values at 10 nm and 20 nm intervals were extrapolated using the **E1** and **E2** methods to the range of (360 nm, 780 nm). The extended reflectance values at 10 nm and 20 nm intervals were used to calculate the tristimulus values later.

Six illuminants (D65, A, D50, F02, F07 and F11) and two CIE (1931/1964) standard observers were used in this study. Five weighting tables at 10 and 20nm intervals were used. They are the optimum weighting tables (**OP**) [4] , the least squares weighting tables (**LS**) [5], the ASTM Intl. standard tables of Tables 5 (**T5**) and 6 (**T6**), and the direct selection (**DS**) tables by taking data directly from 1-nm table.

Three sets of tristimulus values were obtained by using the 1-nm, 10-nm, and 20-nm weighting tables and reflectance values between 360nm and 780nm. The 1nm tristimulus values were taken as standard and were used to compute the CIELAB colour differences ( $\Delta E^*_{ab}$ ) against the 10-nm and 20-nm tristimulus values. Since the colour differences are not normally distributed, the median of the coour differences will well represent the average of the data. Therefore, **median** colour differences were used for measuring the performances of each weighting table and each extrapolation methods. A smaller colour difference indicates a better performance of the weighting table and extrapolation method considered.

## 3. RESULTS AND DISCUSSIONS

It was found that the standard observers have some influences on the performance of each weighting table. However, the overall rankings of all weighting tables do not change with the change of standard observers. Therefore, the results from the two standard observers for each weighting table were combined together. All the results for the 10 nm and 20 nm weighting tables are listed in Tables 1 and 2 respectively, in which OP/E1 means the tristimulus values were calculated using the extrapolation E1 method and the optimum weighting tables. Others have the similar meanings. The values in each row represent the performances of the weighting tables combined with an extrapolation method (indicated by the characters in the first column of the same row) under each of the 6 illuminants. The smaller the value is, the better the combination of the weighting table and extrapolation method performs. The smallest value in each column is presented in bold. For example, the value 0.0021 in the position of fifth row and forth column of Table 1, is the median CIELAB colour difference of the least squares weighting table plus the extrapolation E2 method (LS/E2) under the illuminant D50.

When comparing the two extrapolation methods, it seems that E2 method performed better than E1 method for the OP, LS and T6 types of weighting tables at 10 and 20nm intervals under all 6 illuminants. While, for the T5 and DS weighting tables, E2 method performed better under some illuminants, and not for all other illuminants. However, the two extrapolation methods do not differ too much for all weighting tables. This can be seen from Tables 3 and 4, where median differences of the two extrapolation method for all weighting tables at 10 and 20 nm intervals respectively are listed. The positive values indicate E2 method is better, and negative values indicate the opposite. Thus the results clearly show the difference using the two methods for each weighting tables occurs in the third decimal place in terms of $\Delta E^*_{ab}$ except for T6 table at 20 nm intervals under illuminant F2 where the difference starts from second decimal place.

When comparing different weighting tables, it can be seen that from Table 1 for the 10nm weighting tables, OP and LS tables gave similar performance and performed better than the other weighting tables. The ranking from the best to worst is LS, OP, T6, T5 and DS. Generally speaking, the first four have little difference and performed much better than DS table. Using any of the first four weighting tables may introduce errors (against the CIE standard) being less than 0.009 and 0.05 $\Delta E^*_{ab}$ units under the continuous and fluorescent illuminants respectively. While using DS table will result in errors as large as 0.03 and 9.0 $\Delta E^*_{ab}$ units respectively. The ratios are 3 and 180 respectively, which mean any of the first four tables is 3 times better than DS table under continuous illuminants, and 180 times better under fluorescent illuminants. From Table 2, it clearly shows the ranking from the best to worst is OP, LS, T6, T5 and DS. Roughly speaking, the first three (OP, LS and T6) are the same, then followed by T5, and DS table again performed the worst. Using DS table can result in errors as large as 0.6 and 22 $\Delta E^*_{ab}$ units under the continuous and fluorescent illuminants respectively. While using any of the first four can cause errors are less than 0.07 and 0.3 $\Delta E^*_{ab}$ units under the continuous and fluorescent illuminants respectively. Therefore, any of the first four tables is nearly 9 times better than DS table under continuous illuminants, and 73 under fluorescent illuminants.

| | D65 | A | D50 | F2 | F7 | F11 |
|---|---|---|---|---|---|---|
| OP/E1 | 0.0053 | 0.0049 | 0.0043 | 0.0169 | 0.0143 | 0.0310 |
| OP/E2 | 0.0035 | 0.0034 | 0.0030 | **0.0161** | **0.0139** | **0.0300** |
| LS/E1 | 0.0043 | 0.0039 | 0.0031 | 0.0173 | 0.0142 | 0.0310 |
| LS/E2 | **0.0026** | **0.0030** | **0.0021** | 0.0164 | 0.0140 | **0.0300** |
| T6/E1 | 0.0078 | 0.0061 | 0.0035 | 0.0216 | 0.0162 | 0.0426 |
| T6/E2 | 0.0059 | 0.0046 | 0.0030 | 0.0190 | 0.0152 | 0.0426 |
| T5/E1 | 0.0083 | 0.0088 | 0.0074 | 0.0228 | 0.0165 | 0.0417 |
| T5/E2 | 0.0083 | 0.0079 | 0.0083 | 0.0199 | 0.0153 | 0.0433 |
| DS/E1 | 0.0272 | 0.0260 | 0.0283 | 7.2402 | 4.6035 | 8.9225 |
| DS/E2 | 0.0307 | 0.0253 | 0.0279 | 7.2410 | 4.6063 | 8.9227 |

**Table 1:** Performance of each table at 10nm interval under the combination of the two observers in terms of Median CIELAB colour difference

## 4. CONCLUSIONS

The findings of this study are summarized below:

- For the two extrapolation methods, E2 method performed better than E1 method for OP, LS, and T6 weighting tables. While for T5 and DS tables, some times E1 performed better and some times E2 performed better. The difference between using the two E1 and E2 methods in a particular weighting table occurs in the third decimal place in terms of $\Delta E^*_{ab}$ .
- DS tables performed much different from the others. In general, it cannot be used for computing CIE tristimulus values. It can produce as high as 9 $\Delta E^*_{ab}$ units for the 10 nm tables and 22 $\Delta E^*_{ab}$ units for the 20 nm tables under fluorescent illuminants. While, for OP, LS, and T6 tables, they may produce as high as 0.05 $\Delta E^*_{ab}$ units for the 10 nm tables and 0.3 $\Delta E^*_{ab}$ units for the 20nm tables under fluorescent illuminants. For the continuous illuminants, the differences between DS tables and any of OP, LS, and T6 tables are not that much.
- Comparing the performance of different weighting tables, OP and LS tables performed better than the others. OP and LS tables performed similar under 10 nm interval, while OP tables performed clearly better than LS tables under 20 nm interval. In general, the tables can be ordered from the best to the worst as: OP, LS, T6, T5 and DS. Therefore it is recommended that OP table plus the E2 method should be used for the practical calculations of the CIE tristimulus values.

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| | **D65** | **A** | **D50** | **F2** | **F7** | **F11** |
|---|---|---|---|---|---|---|
| OP/E1 | 0.0147 | 0.0177 | 0.0110 | 0.0526 | 0.0413 | 0.1509 |
| OP/E2 | **0.0132** | **0.0149** | **0.0103** | **0.0462** | **0.0375** | **0.1495** |
| LS/E1 | 0.0162 | 0.0205 | 0.0121 | 0.0545 | 0.0421 | 0.1543 |
| LS/E2 | 0.0161 | 0.0197 | 0.0120 | 0.0535 | 0.0414 | 0.1536 |
| T6/E1 | 0.0152 | 0.0181 | 0.0115 | 0.0817 | 0.0640 | 0.1725 |
| T6/E2 | 0.0134 | 0.0136 | 0.0104 | 0.0700 | 0.0583 | 0.1720 |
| T5/E1 | 0.0626 | 0.0571 | 0.0618 | 0.0985 | 0.0968 | 0.2552 |
| T5/E2 | 0.0654 | 0.0644 | 0.0654 | 0.1042 | 0.1053 | 0.2573 |
| DS/E1 | 0.6026 | 0.1526 | 0.5392 | 3.6964 | 3.1664 | 21.5613 |
| DS/E2 | 0.6050 | 0.1523 | 0.5430 | 3.6941 | 3.1669 | 21.5612 |

**Table 2:** Performance of each table at 20nm interval under the combination of the two observers in terms of Median CIELAB colour difference.

| | D65 | A | D50 | F2 | F7 | F11 |
|---|---|---|---|---|---|---|
| OP | 0.0018 | 0.0015 | 0.0013 | 0.0008 | 0.0004 | 0.0010 |
| LS | 0.0018 | 0.0009 | 0.0009 | 0.0009 | 0.0002 | 0.0009 |
| T6 | 0.0019 | 0.0014 | 0.0005 | 0.0026 | 0.0010 | 0.0001 |
| T5 | ˜0.0000 | 0.0009 | ˜0.0009 | 0.0029 | 0.0012 | ˜0.0016 |
| DS | ˜0.0035 | 0.0007 | 0.0004 | ˜0.0009 | ˜0.0028 | ˜0.0003 |

**Table 3**: Influence of the extrapolation methods for each weighting tables at 10 nm interval

| | D65 | A | D50 | F2 | F7 | F11 |
|---|---|---|---|---|---|---|
| OP | 0.0015 | 0.0028 | 0.0007 | 0.0064 | 0.0038 | 0.0014 |
| LS | 0.0000 | 0.0008 | 0.0001 | 0.0010 | 0.0007 | 0.0007 |
| T6 | 0.0018 | 0.0045 | 0.0011 | 0.0117 | 0.0057 | 0.0006 |
| T5 | ˜0.0028 | ˜0.0073 | ˜0.0036 | ˜0.0057 | ˜0.0085 | ˜0.0021 |
| DS | ˜0.0024 | 0.0003 | ˜0.0039 | 0.0023 | ˜0.0005 | 0.0001 |

**Table 4**: Influence of the extrapolation methods for each weighting tables at 20 nm interval

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