# Experimental Verification of Trichromatic Generalisation

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**ABSTRACT**

A colour matching psychophysical experiment of the maximum saturation type[1a] was conducted on a group of five observers, using two sets of narrow-band primary stimuli and six narrow-band test stimuli. An analysis of the deviations of the results from the assumptions of the Trichromatic Generalisation (TG) was performed taking into account the uncertainty (variability) of the colour matching results. Consistent failures of the TG were observed in the results of a single observer; these failures could be only partially explained by a model of rod intrusion. The mean results of all the observers were found to largely agree with the TG. The reported results are in agreement with previously published works on the subject.

## 1. INTRODUCTION

At the basis of the CIE system of colorimetry lies the Trichromatic Generalisation (TG)[1b], which is a mathematical formulation of Grassmann's laws of additivity[2]. The TG comprises four laws that allow the handling of quantities of coloured stimuli in accordance with standard rules of algebra: Symmetry, Transitivity, Proportionality and Additivity.[1b]

In an additive colour matching experiment, a test stimulus is matched in colour by the mixture of three primary stimuli:

$$\mathbf{Q} = R\mathbf{R} + G\mathbf{G} + B\mathbf{B} \tag{1.1}$$

in which $\mathbf{Q}$ is unit amount of the test stimulus, and its "tristimulus values" $R$, $G$ and $B$ are scalar multipliers of unit amounts of the primaries $\mathbf{R}$, $\mathbf{G}$ and $\mathbf{B}$[1c]. Furthermore, tristimulus values measured with one set of primaries can be transformed to another set of primaries in the same way that a set of coordinates can be transformed from one coordinate system to another, which can be summarised as:

$$\mathbf{T}_{\mathrm{d}p} = \mathbf{M}^{-1}\mathbf{T}_{\mathrm{s}} \tag{1.2}$$

where $\mathbf{T}_{\mathbf{d}p}$ is the 3×1 vector containing the tristimulus values of the stimulus in the primary space **d** (destination), $p$ stands for "predicted", $\mathbf{T_s}$ is the vector of the same size containing the tristimulus values of the same stimulus in the primary system **s** (source), and **M** is the 3×3 matrix containing the tristimulus values of the primary stimuli of the space **d** as measured by means of the primaries **s**. The superscript "-1" represents the matrix inverse operation.

An experiment has been set up to test the TG in two ways. In one, we verify the proportionality law in its strict sense as defined by the Proportionality Law of the TG. In the second, we test the transformability of the tristimulus values are described by eq. (1.2). For the sake of simplicity of notation these two tests are referred to as the "Proportionality Test" and the "Additivity Test" in the rest of this paper.

## 2. METHOD

The rationale for our test of proportionality is that the tristimulus values of a stimulus should be in proportion to the luminance level of the stimulus, as long as the relative spectral power distribution remains the same. In a maximum-saturation colour matching experiment, the same narrow-band colour stimulus was matched at two luminance levels with a luminance ratio of approximately 1/2. If we denote the test stimulus at unit amount as $\mathbf{Q}$, at the full experimental luminance as $\mathbf{Q}_1$, and at the half experimental luminance as $\mathbf{Q}_2$, the two colour matching equations after the normalisation would be:

$$\mathbf{Q} = k_1\mathbf{Q}_1 = k_1R_1\mathbf{R} + k_1G_1\mathbf{G} + k_1B_1\mathbf{B} \tag{1.3}$$

$$\mathbf{Q} = k_2\mathbf{Q}_2 = k_2R_2\mathbf{R} + k_2G_2\mathbf{G} + k_2B_2\mathbf{B} \tag{1.4}$$

where $R_i$, $G_i$ and $B_i$ ($i$ = 1,2) are the tristimulus values of the stimulus $\mathbf{Q}_i$ in the primary system defined by **R**, **G** and **B**, and $k_i$ are the multipliers required to normalise the tristimulus values to unit amount of **Q,** e.g. $k_i = \mathbf{Q}/\mathbf{Q}_i$. The validity of the assumption of proportionality is then tested by testing the null hypothesis that the tristimulus values of stimulus **Q** measured at different luminance levels are the same, e.g.:

$$H_{0R}: \quad k_1R_1 = k_2R_2 \tag{1.5}$$

Similar expressions apply for $G$ and $B$. This null hypothesis is tested by the application of the standard statistical $t$-test, taking into account the uncertainty of each tristimulus value.

In the additivity test, the same stimulus **Q** was matched twice: in the "source" primary system **s,** and in the "destination" system **d.** This resulted in two sets of tristimulus values, $\mathbf{T_s}$ and $\mathbf{T_d}$ respectively. In addition, the tristimulus values of the primary stimuli of system **d** were matched by the primaries of **s**, so that the transformation matrix **M** (eq. (1.2)) could be constructed. The null hypothesis that the tristimulus values can be reliably transformed between systems of primaries is tested by comparing the tristimulus values $\mathbf{T_d}$ measured directly by visual colour matching with the values $\mathbf{T}_{\mathbf{d}p}$ predicted by the model (eq.(1.2)):

$$H_0: \quad \mathbf{T_d} = \mathbf{T}_{\mathbf{d}p} \tag{1.6}$$

Again, this null hypothesis is tested by application of the standard statistical $t$-test, taking into account the uncertainty of each tristimulus value. However, in this case only the uncertainty of values in vector $\mathbf{T_d}$ can be determined directly from the experimental data. The uncertainties of the members of the vector $\mathbf{T}_{\mathbf{d}p}$ must be determined by application of standard methods of uncertainty propagation,[3] taking into account the uncertainties of the tristimulus values $\mathbf{T_s}$ and of the elements of the transformation matrix **M**.

The visual colorimeter[4] provided a vertically-divided bipartite field of 6° diameter; with binocular viewing at a distance of about 1500 mm. Interference filters of 10 nm bandwidth were used for generation of the test and the primary colour stimuli. The photopic luminance values of the test stimuli were in a range of approximately 0.07 to 3.3 cd/m$^2$; corresponding to 3.2 to 125 photopic trolands calculated with the Trezona model of pupil size.[5]

The choice of the primaries was guided by two main considerations: replication of Thornton's results[6] showing additivity failure; and relation of the findings to traditional colorimetry. Hence, the two chosen sets of primary stimuli were close to the Prime Colour[7] (PC) set (603 nm, 530 nm, 451 nm), and the final set of Stiles and Burch[8] in the experiment that led to the 1964 Standard Observer (641 nm, 521 nm, 441 nm) (denoted by T, for traditional). The test stimuli were at 461 nm, 500 nm, 541 nm, 584 nm, 650 nm and 661 nm.* Stimuli at 461 nm, 541 nm and 661 nm, at a luminance level reduced to approximately half, were used for the proportionality test.

Five observers took part in the experiment, and the hypotheses (1.5) and (1.6) were tested for two cases: for a single observer and for the mean results of all observers. Observer B – the first author – performed ten repetitions of every match on separate days, thus providing the data for the "single observer" analysis. The other four observers performed three repetitions each on separate days, and their mean data, together with the mean of observer B, were used for the "all observers" analysis.

Spectroradiometric measurements were taken from the test and the matching fields immediately after the observer pronounced a match. The tristimulus values were then calculated by integrating the energy in the range of ±20 nm from the corresponding filter's peak wavelength, and normalising to 1 W/sr/m$^2$ in the test stimulus.

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* It worth noting that the stimuli at 500 nm, 584 nm and 650 nm – close to those known as "Anti-Prime" colours – were originally planned to be used as an additional set of primaries. However, they were abandoned due to observers' complaints that they are "not intuitive" and are extremely difficult to use in the maximum saturation type of colour matching. Thornton's reported experiment[6], which used the AP lights as primaries, was of Maxwell[1a] type and not of the Maximum Saturation[1a] type.

## 3. RESULTS

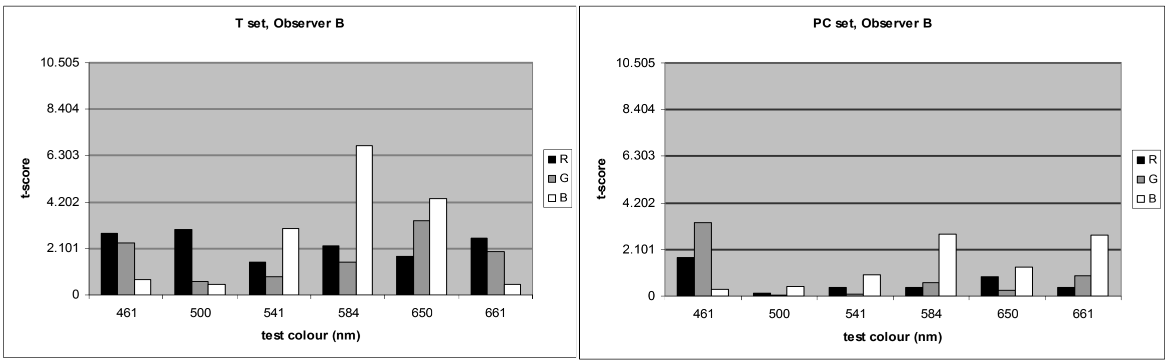
All the reported results are based on the standard statistical 95% confidence *t*-test. For the mean results of all the observers, the TG was found to hold in all but one of the tests: the proportionality test for 661 nm stimulus matched by the PC set primaries. For this stimulus, observers consistently set the Blue tristimulus value to be statistically different (larger) for the stimuli with reduced luminance level. In other words, reduction of the luminance caused all the observers to require relatively more blue light to match the 661 nm stimulus. Some studies[9] suggest that rods can have a blue effect on the perceived colour, and as we show later in this paper there is a significant rod participation in matches of far-red colours with the PC primary set.

The situation is strikingly different in the results for the single observer, where the TG assumptions failed in most of the cases. Using the T primary set as the source set, all the stimuli "failed" in both – the proportionality and the additivity – tests. For the PC primary set, the proportionality test failed for two test stimuli (461 nm and 661 nm) but did not fail for the third (541 nm); the additivity test "failed" in three out of six stimuli. This dependence of the additivity results of the single observer on the spectral position of the primaries is illustrated in Fig. 1.

The reported results agree with previous publications on the subject. In the study that led to the derivation of the 1964 Standard Observer, Stiles[10] reported that the colour matching data of one third of his observers shown "substantial defects in additivity"; the mean results of all the observers, however, were reported to agree with the additivity assumption. Thornton[6] reported that the tristimulus values transformed from the PC set of primaries to other sets agree with the experimentally derived values better then if the transformation is performed in the opposite direction.

The conditions in the experiment – large field and low luminance – are known to favour the participation of the rod receptors in the colour match. In this case, colour vision cannot be considered as strictly trichromatic, hence the validity of the TG cannot be assumed. Therefore, it was of interest to try to relate rod intrusion with the failures of the TG in the single observer's results. An analysis of rod participation[1d] was performed using the Aguilar and Stiles[1e] TVI function and the Trezona[5] model of pupil size. In Fig. 2, the extent of the rod mismatch is plotted against the wavelength of the stimulus. Comparison of the corresponding plots in Fig. 1 and 2 shows only partial correlation between the two sets of data. In both primary sets, significant (>1 and <-1) rod intrusion occurs in all the stimuli in the additivity test, with the exception of 541 nm in PC set. However, in the PC set the results for the stimuli at 500 nm, 650 nm do not show additivity failure - despite the rod intrusion.

A number of possible reasons can be suggested for this phenomenon. One is related to observer metamerism: the luminous efficiency curves of the specific observer in our experiment may differ from the standard ones. Others have to do with the model employed in the evaluation of the rod intrusion: extreme non-uniformity of the scale of the rod participation values;[11] the TVI curve[12,13] may be not applicable to the conditions of the colour matching experiment;[13] the model of pupil size[5] may predict the pupil size of the real observer incorrectly. Finally, there could be additional, non rod-related factors in the colour vision mechanism that cause colour matches to be non-additive.[14,15]




A B

**Figure 1:** Results of the t-test statistics – for each tristimulus value of every test stimulus. The ordinate scale is the 95% critical values from the t-distribution table; bar crossing the first gridline signifies the failure in additivity for the corresponding tristimulus value. **A:** T primary set **B:** PC primary set.

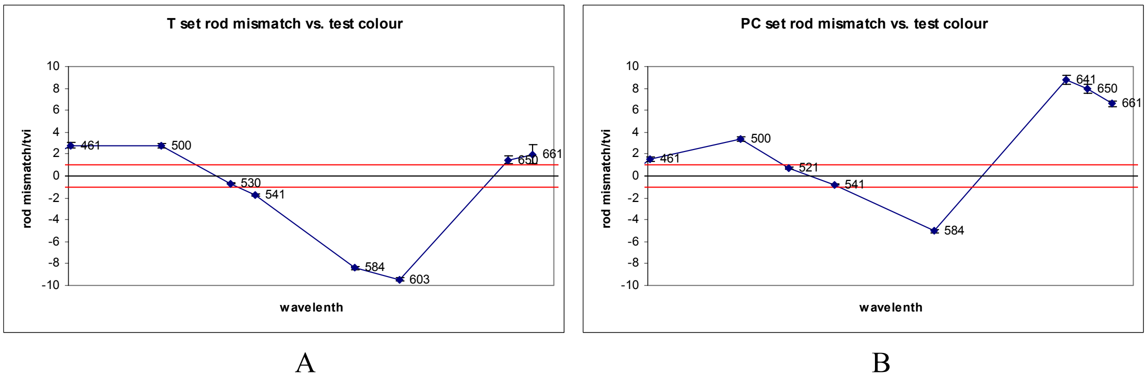



**Figure 2:** Result of the analysis of the rod participation. The ordinate scale is the ratio of the rod mismatch to the threshold-versus-illuminant value, e.g. values larger then 1 and smaller then -1 (horizontal lines) signify significant rod participation in the match. **A:** T primary set **B:** PC primary set.

## 4. CONCLUSIONS

Results of an experimental verification of the Trichromatic Generalisation are presented. In the given experimental conditions, the Generalisation was found to break down in the results of a single observer, but to hold when the average results of all the observers are considered. The TG failures of the single observer depend on the spectral position of the primary stimuli. The failures in additivity in the results of the single observer could be only partially explained by rod participation.

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