# A Flexible Bandpass Correction Method for Spectrometers

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## ABSTRACT

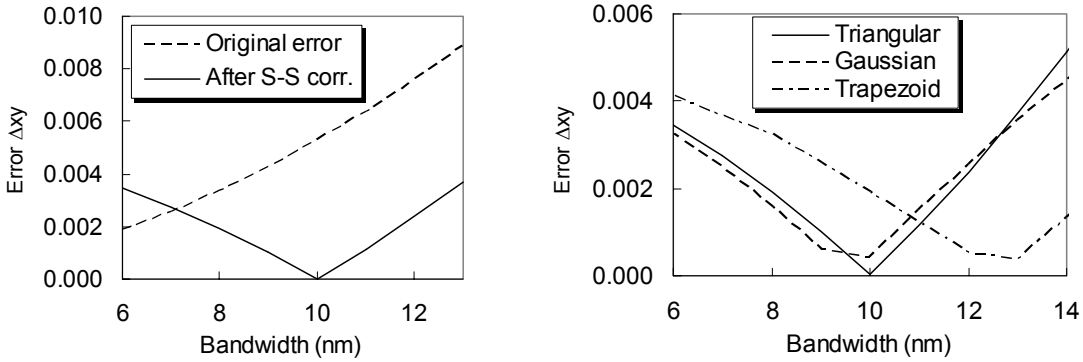
A method for the correction of bandpass errors in spectrometers has been developed. This method is an improvement over the Stearns and Stearns method, which is limited in use to a triangular bandpass function and requires the bandwidth to be matched with the scanning interval. A numerical approach is applied to obtain the relationship between measured values and true spectral values of radiation for an arbitrary bandpass function. The developed method allows for correction of any shape of bandpass function, even with asymmetries, and does not require the bandwidth and the scanning interval of the spectrometer to be matched.

## 1. INTRODUCTION

The bandpass of spectrometers (spectrophotometers and spectroradiometers) can cause significant errors in object color measurements as well as light source color measurements. The errors can be significant if the bandwidth (full width at half maximum) is larger than 5 nm[1]. Even with a 5 nm bandwidth, the errors can be considerable for special samples and for applications that require low uncertainty. For corrections of such errors due to bandpass, a simple method by Stearns and Sterns[2] is available (hereinafter, abbreviated as S-S method), as well as weighting factor tables that incorporate such corrections of bandpass[3]. These methods, however, require that the bandpass function be triangular and that the bandwidth of the spectrometer and the data interval (scanning interval) are matched (e.g., 5 nm scanning interval for a 5 nm or 10 nm bandwidth). While many commercial spectrometers are designed this way, many other instruments do not meet these requirements and the correction cannot be applied. Further, in real spectrometers, the shape of the bandpass is not perfectly triangular and the bandwidth is not perfectly constant throughout the spectral region. These practical limitations make the application of the S-S method difficult or less effective. To solve such problems, an improved method for bandpass error correction, extended from the S-S method, has been developed. A numerical approach is used to obtain the relationship between measured values and true spectral values of radiation for an arbitrary bandpass function. This method works for any shape of the bandpass function, even with asymmetries, and does not require the bandwidth and scanning interval to be matched.

## 2. EXAMINATION OF THE S-S METHOD

The S-S method was developed as an analytical solution of the relationship between measured values and true spectral values of radiation. The solution is obtained for a simple bandpass function, which is a symmetrical triangle, and the bandwidth and scanning interval are required to be matched. In real instruments, this requirement is not perfectly satisfied. It is often seen that the bandwidth of a spectroradiometer, designed to have a constant bandwidth, can vary as much as 20%. In some cases, the bandpass function might resemble a Gaussian function or a trapezoid. To evaluate how sensitive the corrected results would be to deviations from the required matched condition, a simulation was performed using the simulation program described in Ref. 1. This program calculates the convolution of a given light spectrum with a given bandpass function to simulate the spectrometer output, and then it applies the S-S correction to the results. The simulations were performed for measurements of several different light spectra at 1 nm intervals with a varied bandwidth around 10 nm. The S-S correction was applied at each wavelength from the neighboring five points with 10 nm spacing. The chromaticity coordinates (*x, y*) were calculated for the results before and after the correction, and the sum-square error $\Delta xy = \sqrt{\Delta x^2 + \Delta y^2}$ was calculated. The 1 nm scanning interval eliminates the effect of




(a) Triangular bandpass (b) Three different bandpass functions

**Figure 1**: The bandpass errors in chromaticity (*x, y*) for a green LED spectrum after the S-S correction is applied at 10 nm data intervals, for a varied bandwidth.

sampling errors.Figure 1 (a) shows representative results for a green LED spectrum (peak at 503 nm), measured (simulated) with a triangular bandpass at varied bandwidth. It is shown that the error, after the S-S correction (solid curve), is minimized at 10 nm (matched condition), and increases when the bandwidth deviates from 10 nm in either direction. The original errors are reduced to nearly zero at 10 nm bandwidth after S-S correction. However, at 8 nm or 12 nm bandwidth (20 % deviation), the reduction of the error is only about half. Simulations of other spectra including surface color samples and even some discharge lamp spectra yielded very similar curves as Fig.1 (a) though the magnitude of error varies. Fig. 1 (b) shows the errors after the S-S correction when the same green LED is measured with different bandpass functions – triangular, Gaussian, and trapezoidal (the ratio of upper base and lower base is 1:2). The bandwidth is calculated as full-width at half maximum. The results show that, for the trapezoidal bandpass, the matched point shifts significantly, while the effect of the Gaussian function is fairly small. These results demonstrate that the matching condition for the bandwidth and scanning interval is critical for the S-S method to be effective. It should be noted that the S-S method is not designed for discontinuous spectra (such as discharge lamps), but the correction actually works for color measurement though the spectrum is not accurately corrected.

## 3. IMPROVED METHOD

An improved method has been developed that can be applied to any arbitrary bandpass function and does not require the bandwidth and scanning interval of the spectrometer to be matched.

When a spectrometer having a bandpass function $s(\lambda,\lambda_0)$ measures radiation having spectral distribution $S(\lambda)$, as illustrated in Fig. 2, the measured value $M_0$ of the spectrometer at wavelength $\lambda_0$ is given by the integration

$$M_0 = \int s(\lambda, \lambda_0)\, S(\lambda)\, \mathrm{d}\lambda. \qquad (1)$$

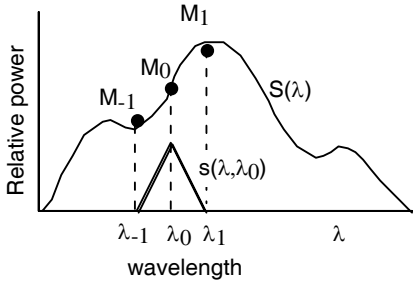



**Figure 2:** An illustration of measurement by a spectrometer.

The same form of equation holds for neighboring scanning points at $\lambda_{-1}$ and $\lambda_1$. The true values of the radiation at the neighboring wavelengths are represented by $S_{-1}$, $S_0$, $S_1$ as

$$S_{-1} = S(\lambda_{-1}),\ S_0 = S(\lambda_0),\ S_1 = S(\lambda_1) \qquad (2)$$

When the bandpass $s(\lambda,\lambda_0)$ encloses $\lambda_{-1}$ to $\lambda_1$, the measured value $M_0$ is related to the true values $S_{-1}$, $S_0$, $S_1$ through Eq.(1) as

$$M_0 = a_{-1} S_{-1} + a_0 S_0 + a_1 S_1. \qquad (3)$$

where $a_{-1}$, $a_0$, $a_1$ are weighting coefficients ( $a_{-1} + a_0 + a_1 = 1$ ). Assuming that $S(\lambda)$ is a smooth function, it is given as a quadratic passing through the three points $S_{-1}$, $S_0$, and $S_1$ as

$$S(\lambda) = f(\lambda; S_{-1}, S_0, S_1). \qquad (4)$$

Figure 3 illustrates a measurement with a spectrometer having an arbitrary bandpass function $s(\lambda,\lambda_0)$ measuring $S(\lambda)$ given as a quadratic. Eq. (1) is restated using the quadratic as

$$M_0 = \int s(\lambda, \lambda_0)\, f(\lambda; S_{-1}, S_0, S_1)\, \mathrm{d}\lambda. \qquad (5)$$

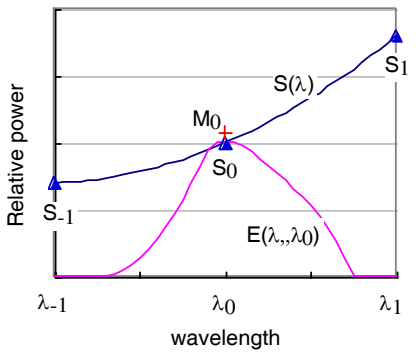



**Figure 3:** Measurement with a spectrometer having an arbitrary bandpass function.

The three coefficients $a_{-1}$, $a_0$, $a_1$ in Eq. (3) can be obtained as the sensitivity of change in $M_0$ when a small change is made in $S_{-1}$, $S_0$, $S_1$. This is the concept of *sensitivity coefficient* used in uncertainty evaluation[4]. The values of $a_{-1}$, $a_0$, $a_1$ are numerically calculated by

$$\begin{aligned} a_{-1} &= \left[ \int s(\lambda, \lambda_0)\, f(\lambda; S_{-1} + \Delta S, S_0, S_1)\, \mathrm{d}\lambda - M_0 \right] / \Delta S \\ a_0 &= \left[ \int s(\lambda, \lambda_0)\, f(\lambda; S_{-1}, S_0 + \Delta S, S_1)\, \mathrm{d}\lambda - M_0 \right] / \Delta S \\ a_1 &= \left[ \int s(\lambda, \lambda_0)\, f(\lambda; S_{-1}, S_0, S_1 + \Delta S)\, \mathrm{d}\lambda - M_0 \right] / \Delta S \end{aligned} \qquad (6)$$

where $\Delta S$ is chosen to be small enough relative to the values of $S(\lambda)$. Once these three coefficients are obtained, simultaneous equations are formed for the five neighboring points $M_{-2}$, $M_{-1}$, $M_0$, $M_1$, $M_2$. With the approximation $S_{-3} = M_{-3}$ and $S_3 = M_3$, as was done in the derivation of the S-S method, the value of $S_0$ (corrected to zero bandwidth) is obtained by

$$\begin{aligned} &S_0 = b_{-2} \cdot M_{-2} + b_{-1} \cdot M_{-1} + b_0 \cdot M_0 + b_1 \cdot M_1 + b_2 \cdot M_2 \\ &\text{with } b_{-2} = \frac{a_{-1}^2}{X},\ b_{-1} = -\frac{a_{-1}}{X},\ b_0 = \frac{a_0}{X},\ b_1 = -\frac{a_1}{X},\ b_2 = \frac{a_1^2}{X}, \text{ and } X = a_0^{\,2} - 2a_{-1}a_1. \end{aligned} \qquad (7)$$

Since this is a numerical approach, the solution can be given for any shape of the bandpass, which may be nonlinear, asymmetric, and/or not matched with the scanning interval, with an assumption that $S(\lambda)$ is a smooth function.

## 4. RESULTS

A computer program has been developed for the method described above. For verification of the method, the five coefficients were calculated for a perfectly triangular bandpass function, whose bandwidth is matched with the scanning interval, and compared with the five coefficients given by the S-S method. Table 1 shows the results of the calculation. The values given by the two methods agree well with only insignificant differences.

**Table 1**: The five coefficients calculated for a triangular bandpass function (10 nm FWHM) with 10 nm scanning interval.

| | S-S method | Ohno method |
|---|---|---|
| $M_{-2}$ | 0.0102 | 0.0101 |
| $M_{-1}$ | -0.1224 | -0.1220 |
| $M_0$ | 1.2245 | 1.2237 |
| $M_1$ | -0.1224 | -0.1220 |
| $M_2$ | 0.0102 | 0.0101 |

For further validation of the method, measurements of several different light spectra were simulated at 10 nm intervals for an asymmetric bandpass function (~7 nm bandwidth) as shown in Fig. 4, and the correction described in this paper was applied to the simulated measurement results at 10 nm intervals. Figure 5 shows an example of a result for a LED spectrum. The figure shows three

curves: the original spectral distribution (solid line), the simulated measured values with a spectrometer (dashed line), and the results after the bandpass correction (dash-dot line). Figure 5 (b) shows that the measured curve was slightly shifted due to the asymmetric bandpass and also the peak is lower due to the bandpass effect. The corrected curve is nearly overlapping with the original curve. The original bandpass error ($\Delta x$, $\Delta y$) in chromaticity coordinates $x$, $y$ was (-0.0007, -0.0153), which was reduced to (0.0001, -0.0004) after the correction (the magnitude of errors reduced to ~1/40). Similar results were obtained for several other light source spectra and various shapes of the bandpass, and it was demonstrated that this correction method works for any shape of the bandpass. As with the S-S method, this method works for color measurement on discharge lamps as well, but note that this method does not correct for sampling errors (associated with the scanning interval), which tend to be serious for sources having very narrow peaks (emission lines) and when the bandwidth and scanning interval are not matched.

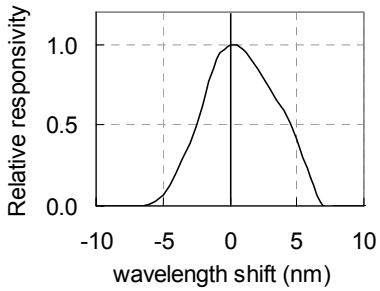
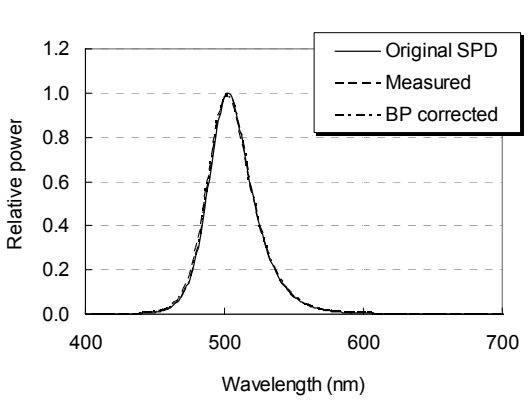
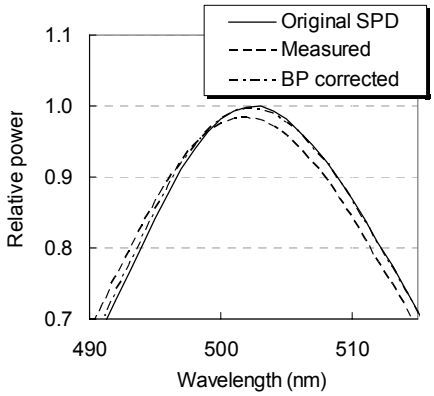



**Figure 4**: The asymmetric bandpass function used for the simulation.




(a) The entire spectral distribution.




(b) Magnification of the peak region.

**Figure 5:** An example of results of correction for the spectrum of a blue-green LED measured with the bandpass shown in Fig.2.

## 5. CONCLUSIONS

An improved method for correction of bandpass errors in a spectrometer has been developed, that can be used for any arbitrary bandpass function and does not require that the bandwidth and scanning interval of the spectrometer to be matched. Simulations using several different light spectra have demonstrated that this method works well for any shape of bandpass under conditions where the bandwidth and the scanning interval are not matched. This method will be useful for object color measurements as well as for light sources including LEDs.

## References

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