Metric tensor to evaluate differences in color attributes

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1. Introduction

In previous works[1,2] a theoretical model of the colour perception process is proposed. This model suggests a generalization of the trichromatic equation in the detection space D_3 (space spanned by the color-matching functions, $e_i(\lambda)$, of the considered observer). In this way, the colour perception process can be interpreted and formalized in this three-dimensional space, which depends on the characteristics of the considered observer. This theoretical model describes the perception of colour, but it does not explain the thresholds (colour, chromaticity, hue, purity, …) experimentally found in colour vision: an observer will have an arbitrary ability to distinguish any two colour stimuli. As it is well known an observer can't distinguish two colour stimuli which are close enough. In this way, it becomes necessary to obtain an adequate theoretical tool in order to explain thresholds in colour perception. It can be done by adding to the previous model the notion of statistical fluctuations in a probability the space.

Any detection process can be explained by analysing the interaction between the detected "object" and the detector. In colour vision, this interaction is produced between the photoreceptors, whose spectral sensitivity is characterized by the colour-matching functions, and the photons contained in a given spectral power distribution $\rho(\lambda)$. In this way, the tristimulus value X_i can be interpreted as the signal provided by the i-th detector when interacting with $\rho(\lambda)$. It is well known that the matter-radiation interaction has a probabilistic nature. This fact will allow us to define the probability p_i that the i-th detector (fundamental mechanism) "sees" the spectral power distribution $\rho(\lambda)$. This probability is proportional to the signal X_i provided by the detector. By characterizing the spectral power distribution in the detection space D_3 by the associated fundamental metamer $n^D(\lambda)$[1,2], this probability can be defined as

$$p_i = \frac{\left\langle e_i^o(\lambda), n^D(\lambda)\right\rangle^2}{\sum_{j=1}^{3}\left\langle e_j^o(\lambda), n^D(\lambda)\right\rangle^2} = \frac{\left(X_i^o\right)^2}{\sum_{j=1}^{3}\left(X_j^o\right)^2} \qquad (i=1, 2, 3) \qquad (1)$$

where $B_e^o = \{e_1^o(\lambda), e_2^o(\lambda), e_3^o(\lambda)\}$ is the orthonormal base obtained from the colour-matching functions of the visual system referred to a new system of primary stimuli $\vec{u}_i^o$ in such a way that X_i^o are the tristimulus values expressed in this system

Probabilities (1) are associated to the interaction between the colour-matching functions and a spectral distribution $\rho(\lambda)$, i.e., each possible colour stimulus has associated its corresponding set of probabilities p_i. In this way, it is possible to represent each colour evoked by an observer in a three-dimensional probability space P_3 by a point ("state") $\vec{p}=(p_1, p_2, p_3)$ instead to do it in the representation system (tristimulus values). In general, probabilities only can be obtained by infinitely repeating an experiment (in our case, to infinitely repeat the interaction of $\rho(\lambda)$ with the visual system). If we perform a probabilistic experiment a finite number N of trials, the frequencies of occurrence $\vec{\xi}=(\xi_1, \xi_2, \xi_3)$ are an approximation to the true probabilities $\vec{p}$. It can be demonstrated[1] that the interaction of a photon with the fundamental mechanisms obeys to the generalized Bernoulli scheme. In this situation, the occurrence frequencies $\vec{\xi}$ in N trials are distributed according to a multinomial distribution. If N is large enough this distribution can be approximated by a Gaussian distribution $\phi(\xi;\vec{p},\vec{\sigma})$, $\vec{\sigma}$ =(σ_1, σ_2, σ_3) being the variances associated with the average values $\vec{p}$ of the statistical populations, i.e., σ_i is the uncertainty in the determination of probability p_i.

In the probability space P_3 two colour stimuli $\vec{p}_1$ and $\vec{p}_2$ have associated the corresponding distributions ϕ_1 and ϕ_2. The distance between these distributions is given by the statistical fluctuations[3] (variances), in such a way that two states $\vec{p}_1$ and $\vec{p}_2$ are distinguishable between them if condition of distinguishability

$$\frac{1}{2}\left[\sum_{i=1}^{3}\frac{(dp_i)^2}{\sigma_i^2}\right]^{1/2} > 1, \qquad (2)$$

is satisfied[4,5]. From this, the statistical distance between two points in the probability space is defined by counting the number of distinguishable states between both points. By introducing a modification of the statistical distance proposed by Wooters[4,5], we obtain the generalized statistical distance[1]

$$ds_g^2 = \frac{1}{4}\left[\sum_{i=1}^{3}\frac{(dp_i)^2}{p_i(1-p_i)}\right]^{1/2}. \qquad (3)$$

When probabilities (1) are introduced in this expression we obtain a metric tensor which allows the measurement of small differences in the colour representation system. By integrating these metric over the geodesic lines in the it is possible to determinate large differences of colour. Metric tensor (3) has been used in order to study the chromaticity thresholds, and we have also determined the curves of wavelength and purity discrimination thresholds[1]. The results obtained are in good agreement with the experimental measurements. The most remarkable characteristics of metric tensor (3) is that it depends on the set of the considered colour-matching functions.

References

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