Inversion of the Spectral Neugebauer Printer Model

A. Alsam, J. Gerhardt, J. Y. Hardeberg
The Norwegian Color Research Laboratory
Gjøvik University College, Norway
Corresponding author: J. Gerhardt (jeremie.gerhardt@hig.no)

ABSTRACT

The Neugebauer equations, both in their classical form and the corresponding modified Yule-Nielsen formulation, are recognized as the best theoretical models to account for halftone printing. Unfortunately, the variables in the equations correspond to the percentage area coverage of the Neugebauer primaries, not the desired printer's digital values. It is thus well understood that solving the inverse Neugebauer equations to yield the colorants densities is not trivial. In this paper we demonstrate that the Neugebauer equations, both in their spectral and colorimetric forms, represent a convex system. We then show that by imposing convexity as a constraint on the solution it is possible to solve the Neugebauer equations in two steps. Firstly, we use a convexly constrained optimization routine to solve for the percentage area coverage and secondly, using the Demichel equations, we solve for the digital values by means of a direct inverse. A clear advantage of our new approach is that the estimated digital values are always feasible. Further, our results show that the proposed formulation yields estimates which are within the device's own variability.

1. INTRODUCTION

In halftone printing, the colour of a surface with a given area is represented as combination of a number of dots with different colours and area proportions. The simplest example for this process is understood in black and white printing, where the print consists of black, white and different shades of grey. In this case black is achieved by covering the printed area with ink, i.e. a 100% coverage, white is achieved by simply not placing ink on the paper, i.e. 0% coverage while any shade of grey is achieved by covering the area partially with the black ink while leaving a percentage of it white, i.e. 0-100% coverage. Mathematically, this process is defined by the Murray-Davies[1] model.

The Neugebauer[2] equation is an extension of the one dimensional Murray-Davies model where the number of colorants used is greater than one. Neugebauer argued that if more than one colorant is used then any color can at best be represented as a linear sum of the Neugebauer primaries. These are defined as the pure colorants and any combination of those. For example, in the commonly employed cyan, magenta and yellow model, the primaries are, cyan, magenta, yellow, the unprinted paper and all respective two and three colour overprint combinations.

It is agreed that the Neugebauer model and its famous Yule-Nielsen[3] extension where the effect of light scattering in the paper is accounted for using a power function; are reasonable[4,5] representations of halftone printing. Still both fail to answer a fundamental question, namely, given the colorimetric value of a colour, which we desire to print: how do we, mathematically, determine the percentage of the different colorants? Said differently, how much cyan, magenta and yellow should we use? Given this question, we define the inverse of the Neugebauer equations as the problem of estimating the densities for a given colorimetric value, xyz, and a set of printer colorants. Furthermore, given the increasing interest in high-fidelity colour reproduction where more than four inks are used to achieve a close spectral match rather than a metameric match[6], we generalise the definition of the Neugebauer inverse problem to the spectral space where we define it as: the problem of finding optimal colorants densities to match an input spectrum.

It is agreed that inverting the Neugebauer equation is a mathematically challenging task. Mahy and Delabatista[7] presented a derivation for an analytical solution to the problem, based on a three inks system. Theoretically, this model[7] can be extended to any number of colorants; however, in practice extending it to a higher number of inks is itself challenging. Furthermore, the authors found that there exists more than one solution to the problem some of which are complex.

To address the difficulty surrounding a theoretical solution for the Neugebauer equation especially for more than three colorants a number of iterative solutions have been proposed[6,8]. To find the actual colorant densities, these methods, normally, rely on large size look-up tables. Further, it is our belief that these methods do not optimally exploit the relation between the Neugebauer equations and the Demichel[9] assumption for the dot placement optimally.

In this paper, we present a two step solution to the Neugebauer inverse equations which can easily be employed in both colour and spectral spaces. Further, the model can very easily be extended to a large number of colorants. The two steps of the proposed model involve solving the Neugebauer equations for the area coverage of the Neugebauer Primaries and then as a second step using those we directly solve for the colorant densities from the Demichel equations. To achieve the first step we first demonstrate that the Neugebauer equations define a convex system. Thus, we solve the equations using a quadratic optimisation routine with convex constrains. A clear advantage of this implementation is that it guarantees that any solution obtained by this formulation is feasible. In the rest of this paper we work on the spectral Neugebauer equation, however, and as previously mentioned the proposed method can be generalised to any space.

2. SPECTRAL NEUGEBAUER PRINTER MODEL

For a reflectance $R(\lambda)$, the spectral Neugebauer (SN) model is defined as:

$$R(\lambda) = \sum_{i=0}^{2^n-1} w_i R_{i,\max}(\lambda). \tag{1}$$

where $R_{i,\max}(\lambda)$ are the Neugebauer Primaries (NP). If n colorants are used the number of corresponding NP is 2^n. The suffix *max* in Equation 1 indicates that all the colorants are printed at maximum digital level, and the weights w_i represent the fractional area[10] covered by the NP. Theoretically, it is assumed that the dot coverage is statistically independent. Further, using the Demichel[9] model it is possible to estimate the weights or fractional area from the printer digital colorant values. For the case of $n = 3$ primaries, the weights are determined by the following equations where $c_i, i = 1,2,3$, are the theoretical colorant values:

$$\begin{aligned} w_{000} &= (1-c_1)(1-c_2)(1-c_3), & w_{001} &= (1-c_1)(1-c_2)c_3, & w_{011} &= (1-c_1)c_2c_3, \\ w_{100} &= c_1(1-c_2)(1-c_3), & w_{110} &= c_1c_2(1-c_3), & w_{111} &= c_1c_2c_3. \\ w_{010} &= (1-c_1)c_2(1-c_3), & w_{101} &= c_1(1-c_2)c_3, & & \end{aligned} \tag{2}$$

3. PROPOSED METHOD FOR THE SN PRINTER MODEL INVERSION

Based on the Demichel equations, all the area coverages w_i have to be positive and less than one. Furthermore, the sum of all w_i has to be equal to one. Given these properties, we can rewrite the classical Neugebauer equation as:

$$R(\lambda) = \sum_{i=0}^{2^n-1} w_i R_{i,\max}(\lambda). \quad \text{subject to} \quad \sum_{i=0}^{2^n-1} w_i = 1 \text{ and } w_i \geq 0 \tag{3}$$

This formulation indicates that the classical Neugebauer equation is convex. Hence we propose an inversion algorithm, which guarantees the feasibility of the solution and which consists of two operations. First we cast the problem of estimating the weights as a convexly constrained quadratic minimisation problem of the form:

$$\min\|r - Rw\|^2 \quad \text{subject to} \quad \sum_{i=0}^{2^n-1} w_i = 1 \text{ and } w_i \geq 0 \tag{4}$$

where r is the spectrum which we wish to print and R is a matrix whose columns are the Neugebauer primaries. Unlike others methods, this formulation is guaranteed to converge to a set of weights, w, which satisfy the intrinsic constraints in the Demichel equations, i.e. $\Sigma_i w_i = 1$ and $0 \leq w_i \leq 1$. Having obtained the weights, we expand Eq. 2 (see Eq. 5) and solve the Demichel inversion problem as a classic linear system where the colorant values are obtained using a direct inversion.

$$\begin{aligned}
w_{000} &= 1 - c_1 - c_2 - c_3 + c_1c_2 + c_1c_3 + c_2c_3 - c_1c_2c_3, \\
w_{100} &= c_1 - c_1c_2 - c_1c_3 + c_1c_2c_3, \\
w_{010} &= c_2 - c_1c_2 - c_2c_3 + c_1c_2c_3, \\
w_{001} &= c_3 - c_1c_3 - c_2c_3 + c_1c_2c_3, \\
w_{110} &= c_1c_2 - c_1c_2c_3, \\
w_{101} &= c_1c_3 - c_1c_2c_3, \\
w_{011} &= c_2c_3 - c_1c_2c_3, \\
w_{111} &= c_1c_2c_3.
\end{aligned} \tag{5}$$

4. EXPERIMENTAL SETUP

For our experiments we used the HP Deskjet 1220c inkjet printer with a CMY *tri-color 78* ink cartridge from HP. The printer was controlled with a driver designed specifically for the task in order to control the dot placement[11]. Moreover, the halftoning was performed by the Floyd-Steinberg algorithm. The prints were made on the *Color Nature 250g/m²* paper from Neusiedler and the spectral reflectances of test charts were measured by GretagMacbeth's Spectrolino spectrophotometer, with a sampling range of 380nm to 730nm with 10nm intervals.

A test chart of 480 different colour patches was printed with CMY inks and measured. Based on the reflectance measurements of this test chart we inverted the SN model with two different methods, namely, the proposed inversion method and a modified Newton Raphson (NR), where the colorant value was modified such that during the iteration process any value greater than 1 or lower than 0 was thresholded to 1 or 0 respectively.

5. RESULTS

The temporal variability of the printer was evaluated by printing the test chart twice in rapid succession. Further, a third test chart was printed and measured a week later. The results were compared with the first prints, see Table 1.

Table 1: The temporal variability of the printer.

Time	Spectral RMS		Rel. $RMS(\%)$		ΔE^*_{ab} (D65)		ΔE^*_{94} (D65)	
	Av.	Max	Av.	Max	Av.	Max	Av.	Max
One day	0.0027	0.0104	0.363	2.077	0.246	0.752	0.301	1.3
One week	0.0173	0.0476	1.926	4.777	1.603	5.014	1.760	4.80

The colorant values obtained for each of methods were used to re-print the test chart. The differences between the original print and the reprints are presented in Table 2.

A dot gain function was applied to the calculated colorant values. This function estimates the printer's digital values from its corresponding effective values. This function is based on the Murray-Davies model. Our results reveal better inversion with our algorithm in every metrics.

Table 2: The relative and perceptual errors based on the original and reprints.

Method	Spectral RMS		Rel. $RMS(\%)$		ΔE^*_{ab} (D65)		ΔE^*_{94} (D65)	
	Av.	Max	Av.	Max	Av.	Max	Av.	Max
NR mod.	0.065	0.283	8.07	26.2	9.01	37.9	9.99	33.03
Proposed	0.034	0.088	5.14	12.3	3.42	12.3	3.94	9.50

In this experiment we have worked on original reflectance spectrum created by our printer. It will be interesting to run our inversion method on spectral data independent of the printer.

6. CONCLUSION

In this paper we presented an inversion method for the Neugebauer equations based on a two steps solution, firstly, we invert the Neugebauer equations to determine the area coverages of the Neugebauer Primaries and as a second step we invert the Demichel equations for the effective colorants values. Finally a dot gain function is used to estimate the printer's digital values.

The deviation of the results from the printer's own repeatability can be explained by the fact that; colour printing is, in general, a complex process to analyse. Further, the process includes aspects which are not accounted for in the theoretical Neugebauer equations, these include: paper, inks and light interaction, as well as the statistical independence of the dot placement assumed in the Demichel equations.

In order to further understand these limitations we indented to make comprehensive comparisons between the proposed algorithm and other research methods. Further, as a future part of this research we plan to expand this model to a general inverse of the modified Neugebauer equations.

References

1. A. Murray. Monochrome reproduction in photoengraving. *J. Franklin Inst.*,**221**:721-744, 1936.
2. H. E. J. Neugebauer. Die theoretischen Grundlagen des Mehrfarbendruckes. *Zeitschrift für wissenschaftliche Photographie Photophysik und Photochemie*, **36**(4) (1937).
3. J. A. C. Yule and W. J. Nielsen. The penetration of light into paper and its effect on halftone reproductions. In *Proceedings of the Technical Association of the Graphic Arts (TAGA)*, volume **3**, p. 65 (1951).
4. D. R. Wyble and R. S. Berns. A critical review of spectral models applied to binary color printing. *Color Research and Application*, 25:4–19, 2000.
5. J. Y. Hardeberg and J. Gerhardt. Characterization of an eight colorant inkjet system for spectral color reproduction. *Proceeding CGIV 2004*, pages 263–267, 2004.
6. L. A. Taplin and R. S. Barns. Spectral color reproduction based on a six-color inkjet output system. *The Ninth Color Imaging Conference*, pages 209–212, November 2001.
7. M. Mahy and P. Delabastita. Inversion of the Neugebauer equations. *Color Research & Application*, 21(6):404–411, December 1996.
8. S. Zuffi and R. Schettini. An innovative method for spectral-based printed characterization. *Color Imaging: Device-independent Color, Color Hardcopy, and Applications VII*, SPIE Proc. 4663, January 2002.
9. M. E. Demichel. *Le procédé*, Vol. 26:17–21, 1924.
10. S.-L. Chang, Y.-T. Liu, and D.-Z. Yeh. A method to estimate fractional areas of Neugebauer primary colors. Proc. *IS&T and SID's Second Color Imaging Conference*, pages 97–100, 1995.
11. J.-J. Sarton. http://xwtools.automatix.de/english/contact.htm.