On the instrumental assessment of illuminant metamerism of parameric sample pairs

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ABSTRACT

The performance of two empirical and a theoretically based correction schemes, intended to minimise the impact of residual colour difference on metamerism assessments of parameric sample pairs, has been tested on a specific sample suite. The Cohen-Kappauf decomposition has been used to separate fundamental stimuli and metameric black contributions for all sample pairs. Results of this ideal metric have been compared to the empirical additive and multiplicative correction schemes used to compensate for residual colour differences present under reference conditions, before evaluating special or general metamerism indices. Of all investigated empirical approaches the multiplicative correction scheme operating in chromaticity colour space exhibits the best performance. All empirical correction models fail completely in case of a mere colour difference and no metamerism. Only the Fairman correction model based on the Cohen-Kappauf decomposition of radiometric functions can resolve the problem and is highly recommended as preparatory step before evaluating any kind of general or special metamerism index.

1. INTRODUCTION

The rematch of a colour shade frequently does not exactly conform to a standard under conditions of a reference or primary illuminant. Magnitude and specific direction of this offshade have certainly an impact on the colour difference under test conditions (secondary illuminant). In order to apply illuminant metamerism calculations to such industrial problems it is imperative to prescribe how to deal with the residual colour difference under reference conditions. Various empirical correction schemes for parameric sample pairs have been proposed in the literature and devised in technical standards such as the *additive* or *multiplicative* correction methods[1-3] intended to minimise the impact of residual colour difference on instrumental metamerism assessments. In the present investigation the performance of such empirical correction methods has been analysed in the light of *Fairman's*[4] proposal of parameric decomposition.

2. THEORETICAL BACKGROUND

The parameric correction takes full advantage of the *Cohen-Kappauf* decomposition of the radiometric functions of a sample pair (standard and batch) into their *fundamental stimuli* and the corresponding *metameric blacks* of both components. In the next step a virtual sample can be formed by adding the metameric black of the sample ($\rho_{mb,spl}$) to the fundamental stimulus of the standard ($\rho_{fs,std}$). Hence, this new virtual sample (ρ^*_{spl}) is corrected for the existing residual colour difference to the standard, while the remaining differences in spectral distribution can only be ascribed to metamerism. This corrected radiometric function can now be compared to the reflectance spectrum of the standard using conventional formulas for metamerism. The resulting formal mathematical construct of the procedure described above is given by

$$\rho^*_{spl} = \mathrm{R}\ \rho_{std} + (\mathrm{I} - \mathrm{R})\ \rho_{spl} = \rho_{fs,std} + \rho_{mb,spl} \quad , \tag{1}$$

where ρ^*_{spl} represents a $n \times 1$ column vector of a corrected radiometric function at the n sampled abscissae values of the used spectrophotometer within the visible spectral range. ρ_{std} and ρ_{spl} denote the spectral radiometric distributions of standard and sample, $\boldsymbol{I}$ the identity matrix, and $\boldsymbol{R}$ *Cohen's* projection matrix.

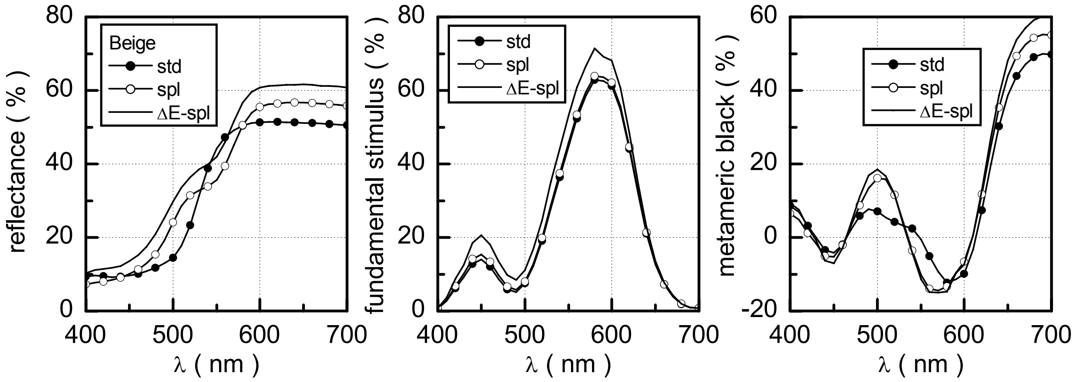

Fig. 1: *Reflectance spectra (left diagram row), fundamental stimuli (middle diagram row), and metameric blacks (right diagram row) of three highly metameric and parameric beige test samples.*

This novel methodical approach can be used to test the performance of empirical parameric correction schemes like the multiplicative and additive correction models, respectively, devised in technical standards. In the latter case the nonzero differences

$$\delta X_r = X_{std,r} - X_{spl,r} \quad , \quad \delta Y_r = Y_{std,r} - Y_{spl,r} \quad , \quad \delta Z_r = Z_{std,r} - Z_{spl,r} \quad , \tag{2}$$

of the original tristimulus values under reference conditions are used to apply the following correction

$$X^{(c)}_{spl,t} = X_{spl,t} + \delta X_r \quad , \quad Y^{(c)}_{spl,t} = Y_{spl,t} + \delta Y_r \quad , \quad Z^{(c)}_{spl,t} = Z_{spl,t} + \delta Z_r \quad . \tag{3}$$

to the chromaticity coordinates under the test illuminant.

In the procedure for the multiplicative correction quotients of the original tristimulus values replace their differences. Hence, the corrected tristimulus values are obtained from

$$X^{(c)}_{spl,t} = A_X \, X_{spl,t} \quad , \quad Y^{(c)}_{spl,t} = A_Y \, Y_{spl,t} \quad , \quad Z^{(c)}_{spl,t} = A_Z \, X_{spl,t} \quad , \tag{4}$$

with the multiplicative factors

$$A_X = \frac{X_{std,r}}{X_{spl,r}} \quad , \quad A_Y = \frac{Y_{std,r}}{Y_{spl,r}} \quad , \quad A_Z = \frac{Z_{std,r}}{Z_{spl,r}} \quad . \tag{5}$$

These corrected values together with $X_{std,t}$, $Y_{std,t}$, and $Z_{std,t}$ are then used to calculate the total colour difference ΔE^* which is used as a measure for the degree of metamerism of two specimens under the test illuminant. Both empirical correction schemes can also be applied directly to coordinates in other colour spaces like *CIELab-76* or *DIN-99*.

3. EXPERIMENTAL DETAILS

In order to study the interference between residual colour difference and metamerism contributions in instrumental assessments in case of parameric sample pairs, a sample suite of varying degree of metamerism and colour difference at a colour centre in the beige colour region of the chromaticity diagram has been prepared experimentally and analysed. The *Cohen-Kappauf* decomposition[5] has been used to separate fundamental stimulus (responsible for colour difference) and metameric black (not actively participating in the colour vision process) contributions for all sample pairs. Hence, the radiometric functions of a given sample pair can be split up in

$$\rho_{std}(\lambda) = \rho_{fs;std}(\lambda) + \rho_{mb;std}(\lambda) \quad , \tag{6a}$$

$$\rho_{spl}(\lambda) = \rho_{fs;spl}(\lambda) + \rho_{mb;spl}(\lambda) \quad , \tag{6b}$$

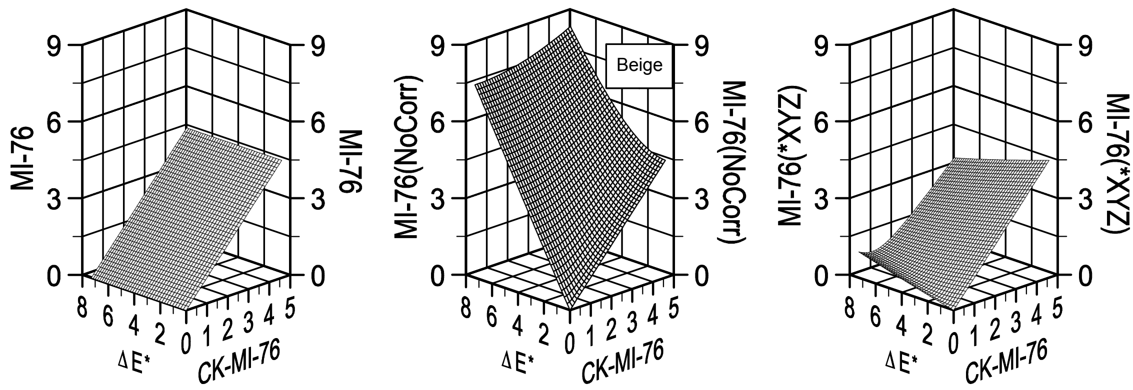

Fig. 2: *Reflectance spectra (left diagram row), fundamental stimuli (middle diagram row), and metameric blacks (right diagram row) of three highly metameric and parameric beige test samples.*

Using these individual contributions of both specimens, a new virtual sample

$$\rho^*_{spl}(\lambda) = (1-x)\rho_{fs,std}(\lambda) + x\,\rho_{fs,spl}(\lambda) + (1-y)\rho_{mb,spl}(\lambda) + y\,\rho_{mb,std}(\lambda) \quad . \tag{7}$$

can be constructed, where x denotes the amount of fundamental stimulus of the sample and y the contribution of the metameric black of the standard. The limiting case $x=0$ corresponds to virtual samples of zero colour difference to the standard (genuine metameric pairs).

4. RESULTS AND DISCUSSION

As described above the *Cohen-Kappauf* projection matrix ***R*** decomposes any radiometric function into its fundamental stimulus and metameric black. Fig. 1 demonstrates that for the investigated metameric pair of solid beige colours their fundamental stimuli closely resemble each other, while their metameric black contributions are markedly different. By comparison, the fundamental stimulus of the parameric specimen plotted in the same diagram presents a sizable difference. However, the metameric black contributions of both metameric and parameric specimens with respect to the standard exhibit the same structure and almost match.

For simulation purposes a special metamerism index (*MI*) has been used which is oriented at its purpose: it should serve as an indication of metamerism of sample pairs matching under daylight conditions (standard illuminant D65), if they are viewed under other secondary light sources (as, e. g., incandescent light). The natural choice of special metamerism index is just the colour difference between a sample pair when viewed under the secondary illuminant.

Fig. 2 (left diagram) depicts a graph for the ideal situation, where residual colour difference and metamerism contributions for pairs of beige parameric samples can be uniquely separated. The surface of the calculated metamerism as a function of residual colour difference and decomposed metamerism resembles that of an ideally, structureless plane with an inclination depending on the metamerism properties of the particular sample pair. There is no interference between both measures describing the colouristic status of the sample pair, i. e., when keeping the metamerism constant and increasing the residual colour difference, the evaluated metamerism index will perfectly stay constant. This ideal behaviour can not be preserved in real examples, when replacing the ideal metric by the multiplicative or no correction scheme. Fig. 2 also depicts such a real example for the same beige parameric sample suite, where the *CIELab-76* metamerism index has been calculated as a function of residual colour difference and *Cohen-Kappauf*-decomposed metamerism without applying any correction scheme (middle diagram) and using the multiplicative correction applied to the triple of chromaticity coordinates. A cursory glance of Fig. 2 reveals that there are strong deviations between the ideal metric (middle diagram) and the uncorrected data set (left diagram), while the multiplicative correction (right diagram) significantly improves the metric resulting in a much better resemblance of the data set to the ideal metric. Furthermore, it is also obvious from Fig. 2 that there is a systematic weakness of the metric for sample pairs differing only by their fundamental stimuli while exhibiting the same residuals.

Another important observation that deserves a mention at this stage is that a comparison of the additive and multiplicative correction schemes, when applied to chromaticity and *CIELab-76* coordinates, clearly leads to the conclusion that the multiplicative correction in chromaticity colour space exhibits the highest robustness concerning the impact of residual colour difference. The additive correction applied to the same colour space does not mitigate the interference between colour difference and metamerism and exhibits the worst performance. In this sense the change of the *DIN* recommendation from the multiplicative[1] to the additive[2] correction method does not find a later theoretical justification. Both correction methods lead to suboptimal results when applied to *CIELab-76* coordinates.

In the analysis described above, a special metamerism index has been chosen for testing the performance of proposed correction schemes. Such indices are known to be in better conformity with visual assessments than general metamerism indices whose metric is based on spectral differences[4] $\Delta\rho = \rho_{mb,std} - \rho_{mb,spl}$. However, in both cases not necessarily a good correlation between visual and instrumental assessments of parameric sample pairs has to be expected, since the colour perception process can not uniquely separate colour difference and metamerism contributions.

5. CONCLUSIONS

Thorough analysis of the results obtained for parameric sample suites of a beige solid colour shade has revealed that of all investigated empirical models the multiplicative correction scheme operating in chromaticity space exhibits the best performance. All empirical correction models fail completely in case of a mere colour difference and no metamerism. Only the *Fairman* correction model based on the *Cohen-Kappauf* decomposition of radiometric functions can resolve the problem and is highly recommended as a preparatory step before evaluating any kind of special and general metamerism index. An extension of the present investigation to other colour centres corroborates the generality of the above-mentioned experimental findings and the conclusions drawn from them. These results will published elsewhere.

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