Reflectance estimation with uncertainty

P. Morovic and G. D. Finlayson
School of Computing Sciences, University of East Anglia
NR4 7TJ, Norwich (UK)
Corresponding author: P. Morovic (peter@cmp.uea.ac.uk)

ABSTRACT

We present a new approach to the problem of estimating the reflectance of a surface, given its RGB, assuming only knowledge of the illuminating light source and the spectral sensitivities of the recording device. Our approach is different from previous ones in that we first solve the most general form of the recovery problem: we calculate the set of all possible surface reflectances, the *metamer set*, and then, at a second stage, we choose a member from this set given additional objectives.

Experiments were carried out to compare the presented model to existing methods. Our results show that metamer set based reflectance estimation consistently outperforms traditional approaches. Moreover, our framework provides a means to express the robustness of the estimate in the form of error-bars.

1. INTRODUCTION

Typical digital colour input devices capture three measurements of a surface reflectance, known as the RGB. Surface reflectance however is a continuous function of wavelength independent both of the device and of the light source. Therefore, a RGB is a limited representation of the surface reflectance in terms of its colour rendering properties. In this paper we address the problem of how to use device RGBs, in order to arrive at the original surface reflectance.

Given an RGB of a surface from a device with known spectral sensitivities captured under a known illuminant it is, however, non-trivial to estimate the spectral reflectance. The reason for this is metamerism: the phenomenon whereby multiple surfaces induce an identical RGB under fixed viewing conditions. In literature the general approach to reflectance estimation is to avoid metamerism by establishing one-to-one relations between RGBs and reflectances, such as a least-squares approach[1] or a three dimensional linear model approach[2]. The disadvantage of these methods is twofold: firstly, their accuracy is limited due to the strong constraints that enforce this one-to-one relation; secondly, the absence of a notion of how good the estimate is, given that in theory any metamer could be correct.

We propose a new 2-step approach to reflectance estimation. First we characterise the entire set of possible candidate reflectances, the metamer set. Secondly we optimise over this set to find a suitable, representative surface as our estimate.

We start by considering the mathematics of how an image is formed on the imaging plane of a digital colour input device. For a flat Lambertian surface $S(\lambda)$, uniformly illuminated by a light source $E(\lambda)$ captured by a linear device whose i-th spectral sensitivity is $R_i(\lambda)$, it's i-th response ρ_i is proportional to:

$$\rho_i = \int_{\omega} S(\lambda)E(\lambda)R_i(\lambda)d\lambda \qquad (1)$$

where ω is the interval of visible wavelengths λ. Given $E(\lambda)$ and $R_i(\lambda)$ (where normally $i = 1, 2, 3$ and the vector $\underline{\rho}$ corresponds to the RGB) it can be shown that there are many possible surfaces with different reflectance functions $S(\lambda)$ that result in the same RGB. This phenomenon is known as metamerism and is due to the under-determined nature of Eq. (1), mapping continuous functions (reflectances) to 3 dimensional vectors (RGBs).

2. METAMER SETS

Solving for the set of all *$S(\lambda)$*'s means inverting Eq. (1). This can be done by a simple decomposition of the reflectance into a *fundamental* part $S_\rho(\lambda)$ (accounting for the actual RGB) and a *metameric black* part $S_0(\lambda)$ (inducing an RGB of zeros). Since the metameric black part can be arbitrarily scaled, as $\alpha 0 = 0$, this decomposition forms an unbounded, infinite, convex set. This set, however, will contain functions that do not satisfy properties of real, natural reflectances found in the world.

So, we introduce a convex set of constraints on plausible reflectances, denoted ***P***: *physical realisability* (all values must lie between 0 and 1, since no surface can reflect less than no light and no surface can reflect more than all light. Note that we do not consider fluorescent surfaces, that can violate this latter condition), *smoothness* (we represent surface reflectances in a 5 to 8 dimensional linear model basis) and *naturalness* (we define surfaces to be natural, if they can be created as a convex combination of existing measured surface reflectances).

The above decomposition together with the constraints in ***P*** then defines the metamer set $M(\underline{\rho})$[5], the set of all plausible surface reflectances that could have induced the response $\underline{\rho}$.

3. REFLECTANCE ESTIMATION

Of course, in reflectance estimation, we wish to identify a single surface as our estimate, so we investigate three methods to choose from the metamer set.

The maximum ignorance approach to such a choice is that of choosing an estimate mitigating worst case error, the *centroid* of the metamer set (MSC). The centroid is simply that point within a set, which minimises the maximum possible distance to any other point within this set.

$$\hat{S}(\lambda) = \min_{S_i(\lambda)\in M(\underline{\rho})} \max_{S_j(\lambda)\in M(\underline{\rho})} \left\| S_i(\lambda) - S_j(\lambda) \right\| \tag{2}$$

The choice of centroid of course implicitly assumes a uniform probability distribution of surface reflectances.

Freeman and Brainard[3] argued that surface reflectances follow a truncated normal distribution in the domain of linear model weights. So, an alternative way of choosing a reflectance is to adopt this assumption and maximise this probability:

$$\hat{S}(\lambda) = \max_{S(\lambda)\in M(\underline{\rho})} P(S(\lambda)) \tag{3}$$

This method is also known as *maximum likelihood* (ML).

Finally we propose to choose the smoothest reflectance in the metamer set (MSS). Numerous authors have argued that smoothness is an inherent property of surface reflectances (e.g. van Trigt[4]). Maximising a measure of smoothness, such as the integral of the square of the first derivative of a reflectance, we can solve for the smoothest metamer.

$$\hat{S}(\lambda) = \max_{\mathrm{S}(\lambda)\in \mathrm{M}(\underline{\rho})} \int_{\omega} \left(\frac{\partial S(\lambda)}{\partial \lambda} \right)^2 \tag{4}$$

While in each of these methods we choose a single surface to represent the entire set, the set itself is a measure of goodness of the estimate. The larger this set, the larger the potential error and the bigger the differences between the choices, and vice versa.

4. RESULTS

We compare the new, metamer set based methods to standard approaches known in literature, such as the 3D linear model weight estimation method, linear least squares (using higher dimensional linear models) and van Trigt's smoothest reflectance functions.

To evaluate the performance of the reflectance estimation techniques, we use two error metrics. If $S_1(\lambda)$ is the original reflectance and $S_2(\lambda)$ the estimate, then the estimation error Δ is:

$$\Delta = \left\| S_1(\lambda) - S_2(\lambda) \right\|_2 \quad (5)$$

that is the L2 Euclidean distance metric in the space of reflectances. The smaller this measure the closer the two reflectances are (two reflectances are identical if $\Delta = 0$). Table 1 summarises the results.

Since the metamer set defines all potential estimates, assuming the conditions are met for the original reflectance to be contained in the metamer set, we have a definition of uncertainty. Therefore we can examine the worst case error Δ_T, that is the distance between the selected reflectance and the furthest possible metamer.

$$\Delta_T = \max_{S_i(\lambda) \in M(\underline{\rho})} \left\| \hat{S}(\lambda) - S_i(\lambda) \right\|_2 \quad (6)$$

Due to metamerism, any of the reflectances in the metamer set could be the sought original, thus the furthest to our particular choice represents an expression of the stability of each of the methods.

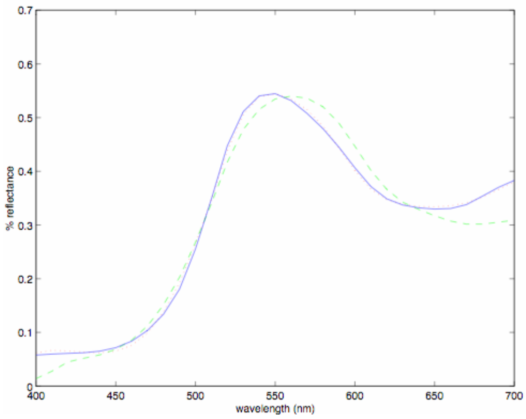

Figure 1: An example of reflectance estimation performance. The solid line shows the original, the dashed line shows an estimate with $\Delta = 0.0103$, and the dotted line shows an estimate with $\Delta = 0.0052$.

Table 1 summarises results for estimating the surface reflectances of the Macbeth ColorChecker Chart (24 coloured samples) given their RGBs under illuminant D65 for a camera with known spectral sensitivities (Figure 1).

Table 1: Reflectance estimation statistics for the Macbeth ColorChecker Chart taken under CIE illuminant D65. The top half of the table shows mean and maximum actual Δ errors, while the bottom half shows mean and maximum worst case Δ_T errors.

	LSQ	3D	SM	MSC	MSS	ML
Mean Δ	0.0055	0.0058	0.0068	0.0042	*0.0039*	*0.0039*
Max Δ	0.0116	0.0110	0.0181	*0.0084*	0.0094	*0.0077*
Mean Δ_T	0.0094	0.0094	0.0113	*0.0052*	0.0071	*0.0065*
Max Δ_T	0.0158	0.0155	0.0200	*0.0126*	0.0158	*0.0142*

The error statistics show that using metamer set based estimation results in a significant gain in performance. Mean Δ is reduced by 30% for the maximum likelihood (ML) and smoothest reflectance within metamer set (MSS) selection methods, and the centroid (MSC) is second-best in the Δ statistics. We also see that the smoothest reflectance without constraints (SM) performs poorly compared to all remaining methods. To understand the meaning of the Δ values, we plot an example of reflectance estimation in Figure 2.

Perhaps a more robust comparison of the methods is the worst case error metric Δ_T, as it counts with the uncertainty in the estimation process. In the bottom half of Table 1 a similar behaviour of the metamer set methods compared with the traditional ones is apparent, however metamer set centroid (MSC) results in smallest error, followed by the maximum likelihood choice (ML). This is expected, since the centroid is a minimisation in the worst case sense. An interesting observation is also the fact that the magnitude of Δ_T for MSC is smaller than the actual error Δ of any of the traditional methods.

5. CONCLUSIONS

In this paper we have introduced a new approach to estimating the spectral reflectances of surfaces, given their RGBs captured under known conditions. We proposed a two-stage approach in which we first characterise the entire set of possible reflectances, the metamer set. Next we choose one surface reflectances from within this set. This choice of a single metamer is made based on further assumptions on reflectances, such as their probability (normal or uniform) or their inherent smoothness.

We conducted a small set of experiments comparing the new approaches to traditional ones. The error statistics show that using metamer set based estimation results in a significant gain in precision. Mean accuracy is increased by an order of 30% for metamer set based methods. Furthermore, metamer set based methods by definition contain a notion of uncertainty, the metamer itself.

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